Theory and Phenomenology of neutrino oscillations: Lecture II

GIF School
LES NEUTRINOS

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What have we learnt yesterday?

- We have looked at the basic picture of neutrino oscillations and how to compute neutrino oscillation probabilities in vacuum for 2- and 3- neutrino mixing.

- We have discussed the properties (including CP-violation) and implications of neutrino oscillations.

- We have mentioned some more subtle issues in neutrino oscillations (mass, mixing and coherence).

What will you learn from this lecture?

- The effects of matter in neutrino oscillations:
  in particular the enhancement of oscillations due to resonances

- Which oscillations probabilities are relevant in the various experiments and therefore which parameters are they sensitive to?

- The impact of neutrino oscillations in cosmology

Plan of lecture II

- Matter effects in neutrino oscillations
  - Matter potential
  - 2-neutrino oscillations in constant density
  - 2-neutrino oscillations in varying density and the MSW effect for solar neutrinos
  - 3-neutrino oscillations in matter for LBL
  - other effects

- Neutrino oscillations in experiments: linking the theory to experimental situations

- Neutrino oscillations in cosmology: the case of sterile neutrinos

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Neutrino oscillations in matter

- When neutrinos travel through a medium, they interact with the background of electron, proton and neutrons and acquire an effective mass.

- This modifies the mixing between flavour states and propagation states and the eigenvalues of the Hamiltonian, leading to a different oscillation probability w.r.t. vacuum.

- Typically the background is CP and CPT violating, e.g. the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations are CP and CPT violating.

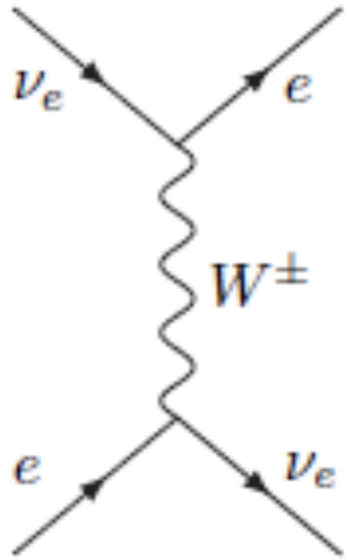
Effective potentials

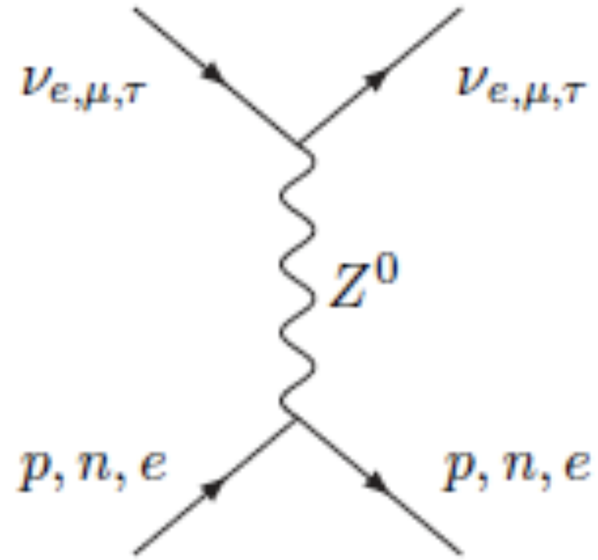
Inelastic scattering and absorption processes go as G_F^2 and are typically negligible. Neutrinos undergo also forward elastic scattering, in which they do not change momentum. [L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); ibid. D 20, 2634 (1979), S. P. Mikheyev, A. Yu Smirnov, Sov. J. Nucl. Phys. 42 (1986) 913.]

As typically the interactions we are interested in happen at energies much below the electroweak scale, we can start with the four-fermion interaction lagrangian

$$\mathcal{L}_{4-f} = -2\sqrt{2} G_F (\bar{\nu}_{eL} \gamma^\rho \nu_{eL})(\bar{e}_L \gamma_\rho e_L) + \cdots$$

Electron neutrinos have CC and NC interactions, while muon and tau neutrinos only the latter.





For a useful discussion, see E. Akhmedov, hep-ph/0001264; A. de Gouvea, hep-ph/0411274.

We treat the electrons as a background, averaging over it and we take into account that neutrinos see only the left-handed component of the electrons.

$$\langle \bar{e}\gamma_0 e\rangle = N_e \quad \langle \bar{e}\vec{\gamma} e\rangle = \langle \vec{v}_e\rangle \quad \langle \bar{e}\gamma_0\gamma_5 e\rangle = \langle \frac{\vec{\sigma}_e \cdot \vec{p}_e}{E_e}\rangle \quad \langle \bar{e}\vec{\gamma}\gamma_5 e\rangle = \langle \vec{\sigma}_e\rangle$$

For an unpolarised at rest background, the only term is the first one. N_e is the electron density.

The Dirac equation (neglecting the mass for simplicity) is

$$\left(i\partial^\rho\gamma_\rho - \sqrt{2}G_F N_e \gamma_0\right)|\nu_e\rangle = 0$$

The neutrino dispersion relation can be found by solving with plane waves, in the ultrarelativistic limit

$$E \simeq p \pm \sqrt{2}G_F N_e$$

For neutrinos and antineutrinos different sign!
The new term is called the matter potential.
Including both CC and NC ones has

medium	$A_{\rm CC}$ for $\nu_e, \bar\nu_e$ only	$A_{\rm NC}$ for $\nu_{e,\mu,\tau}, \bar\nu_{e,\mu,\tau}$
$e, \bar e$	$\pm\sqrt{2}G_{\rm F}(N_e - N_{\bar e})$	$\mp\sqrt{2}G_{\rm F}(N_e - N_{\bar e})(1-4s^2_{\rm W})/2$
$p, \bar p$	0	$\pm\sqrt{2}G_{\rm F}(N_p - N_{\bar p})(1-4s^2_{\rm W})/2$
$n, \bar n$	0	$\mp\sqrt{2}G_{\rm F}(N_n - N_{\bar n})/2$
ordinary matter	$\pm\sqrt{2}G_{\rm F}N_e$	$\mp\sqrt{2}G_{\rm F}N_n/2$

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The Hamiltonian

Let's start with the vacuum Hamiltonian for 2-neutrinos

$$i\frac{d}{dt}\begin{pmatrix}|\nu_1\rangle \\ |\nu_2\rangle\end{pmatrix} = \begin{pmatrix}E_1 & 0 \\ 0 & E_2\end{pmatrix}\begin{pmatrix}|\nu_1\rangle \\ |\nu_2\rangle\end{pmatrix}$$

Recalling that $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$, one can go into the flavour basis

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix}|\nu_\alpha\rangle \\ |\nu_\beta\rangle\end{pmatrix} &= U\begin{pmatrix}E_1 & 0 \\ 0 & E_2\end{pmatrix}U^\dagger\begin{pmatrix}|\nu_1\rangle \\ |\nu_2\rangle\end{pmatrix} \\ &= \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_\alpha\rangle \\ |\nu_\beta\rangle\end{pmatrix} \end{aligned}$$

We have neglected common terms on the diagonal as they amount to an overall phase in the evolution.

The full Hamiltonian in matter can then be obtained by adding the potential terms, diagonal in the flavour basis. For electron and muon neutrinos

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

For antineutrinos the potential has the opposite sign.

In general the evolution is a complex problem but there are few cases in which analytical or semianalytical results can be obtained.

2-neutrino case in constant density

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}$$

If the electron density is constant (a good approximation for oscillations in the Earth crust), it is easy to solve. We need to diagonalise the Hamiltonian.

- Eigenvalues:

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}$$

- The diagonal basis and the flavour basis are related by a unitary matrix with angle in matter

$$\tan(2\theta_m) = \frac{\frac{\Delta m^2}{2E}\sin(2\theta)}{\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e}$$

It can also be rewritten as

$$\sin^2(2\theta_m) = \frac{\left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}{\left(\frac{\Delta m^2}{2E}\cos(2\theta) - \sqrt{2}G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\sin(2\theta)\right)^2}$$

Three interesting limits:

- If $\sqrt{2}G_F N_e \ll \frac{\Delta m^2}{2E}\cos 2\theta$, we recover the vacuum case and

$$\theta_m \simeq \theta$$

- If $\sqrt{2}G_F N_e \gg \frac{\Delta m^2}{2E}\cos(2\theta)$, matter effects dominate and oscillations are suppressed.

- If $\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta$: resonance and maximal mixing

$$\theta_m = \pi/4$$

- The resonance condition can be satisfied for
  - neutrinos if $\Delta m^2 > 0$
  - antineutrinos if $\Delta m^2 < 0$

And conversely, if there is an enhancement in one of the two channels, it is possible to deduce the sign of Δm^2.

- The oscillation probability can be obtained as in the two neutrino mixing case but with

$$\theta \quad \rightarrow \quad \theta_m$$

$$(m_2^2 - m_1^2)/(2E) \quad \rightarrow \quad E_A - E_B$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \sin^2(2\theta_m) \sin^2 \frac{(E_A - E_B)L}{2}$$

2-neutrino oscillations with varying density

Let's consider the case in which N_e depends on time. This happens, e.g., if a beam of neutrinos is produced and then propagates through a medium of varying density (e.g. Sun, supernovae).

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

At a given instant of time t, the Hamiltonian can be diagonalised by a unitary transformation as before. We find the instantaneous matter basis and the instantaneous values of the energy. The expressions are exactly as before but with the angle which depends on time, $\theta(t)$.

We have

$$|\nu_\alpha\rangle = U(t)|\nu_I\rangle, \qquad U^\dagger(t)H_{m,fl}U(t) = \mathrm{diag}(E_A(t), E_B(t))$$

Starting from the Schroedinger equation, we can express it in the instantaneous basis

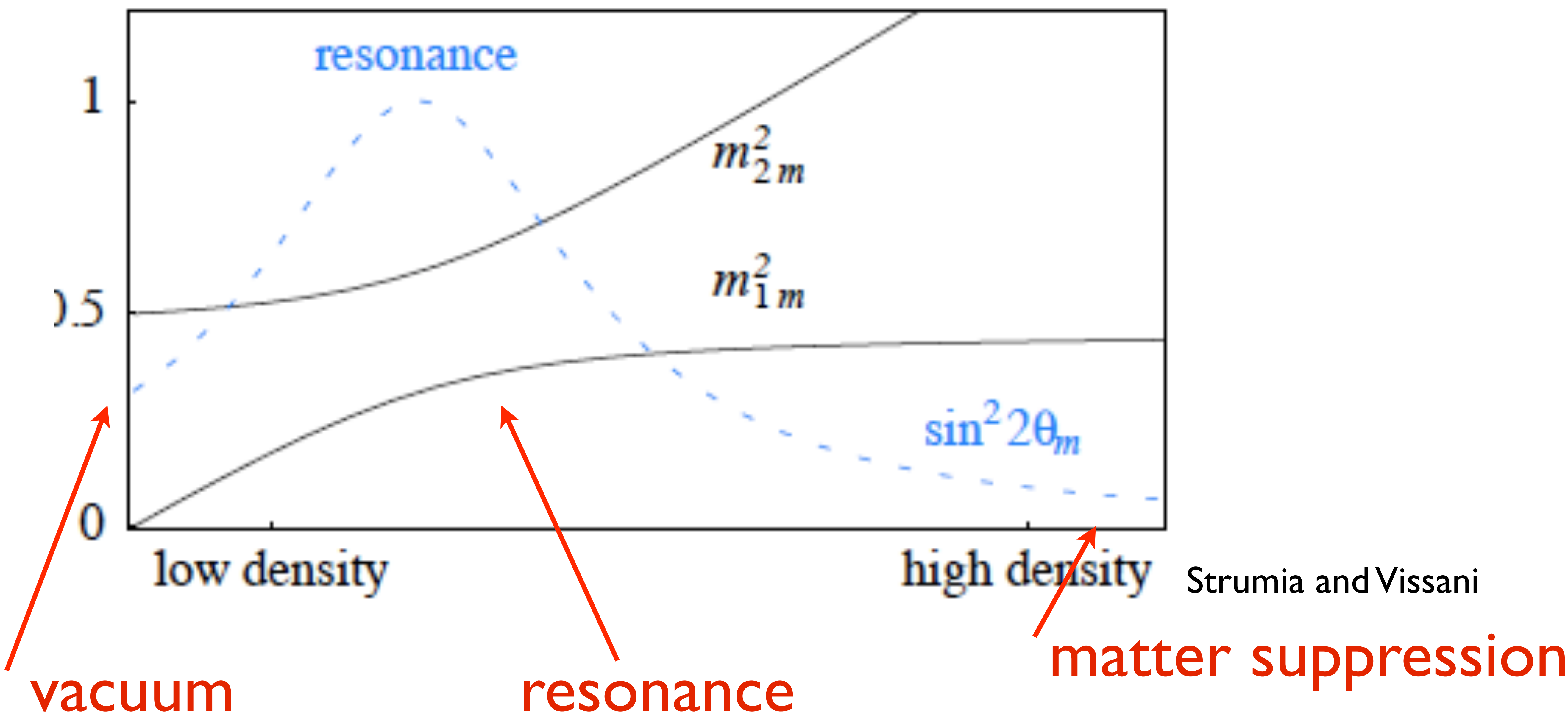
$$i\frac{d}{dt}U_m(t)\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e(t) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} U_m(t)\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix} = \begin{pmatrix} E_A(t) & -i\dot{\theta}(t) \\ i\dot{\theta}(t) & E_B(t) \end{pmatrix}\begin{pmatrix} |\nu_A\rangle \\ |\nu_B\rangle \end{pmatrix}$$

The evolution of ν_A and ν_B are not decoupled. In general, it is very difficult to find analytical solution to this problem.

Going from high density to vacuum, the energies evolve. At the resonance the mixing angle is maximal.

In the adiabatic case, each component evolves independently. In the non adiabatic one, the state can "jump" from one to the other.



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Adiabatic case

If the evolution is sufficiently slow (adiabatic case):

$$|\dot{\theta}(t)| \ll | E_A - E_B|$$

and we can follow the evolution of each component independently.

Adiabaticity condition

$$\gamma^{-1} \equiv \frac{2|\dot{\theta}|}{|E_A - E_B|} = \frac{\sin(2\theta)\frac{\Delta m^2}{2E}}{|E_A - E_B|^3}|\dot{V}_{CC}| \ll 1$$

In the Sun, typically we have

$$\gamma \sim \frac{\Delta m^2}{10^{-9}\mathrm{eV}^2}\frac{\mathrm{MeV}}{E_\nu}$$

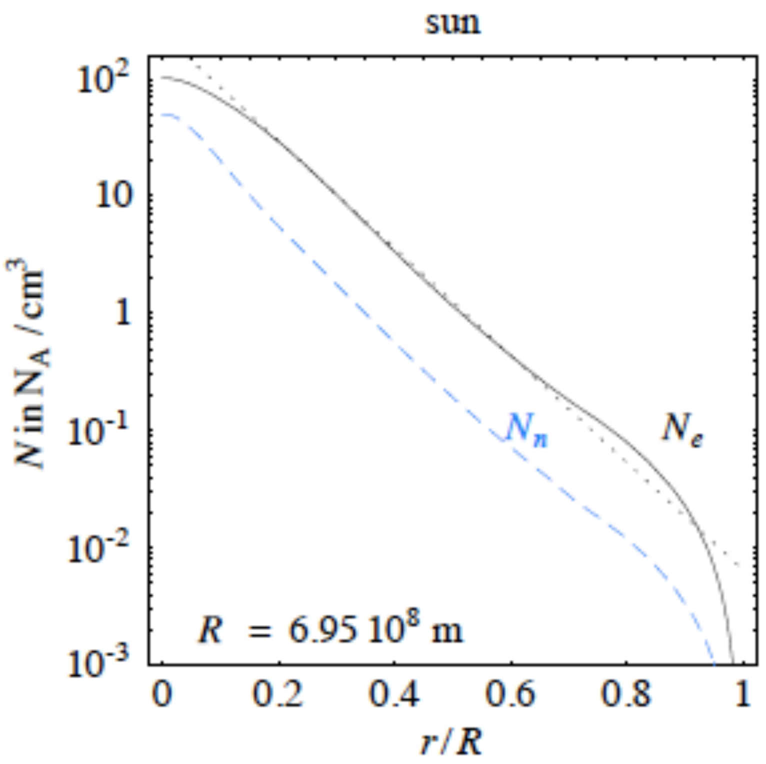
and adiabaticity applies up to ~10 GeV energies.

Beyond adiabaticity

Violations of adiabaticity can be described by the probability of ν_A to jump to ν_B and viceversa: P_c.

For an exponential profile as in the Sun

$$V_{CC} \propto N_e \propto \exp(-r/r_0)$$



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one can find the "jump probability"

$$P_C = \frac{e^{\tilde{\gamma}\cos^2\theta} - 1}{e^{\tilde{\gamma}} - 1}, \qquad \text{with} \quad \tilde{\gamma} \equiv \frac{\pi r_0 \Delta m^2}{E_\nu}$$

Solar neutrinos: MSW effect

The oscillations in matter were first discussed in L. Wolfenstein, S. P. Mikheyev, A. Yu Smirnov.

- Production in the center of the Sun:

matter effects dominate at high energy, negligible at low energy.

The probability of ν_e to be

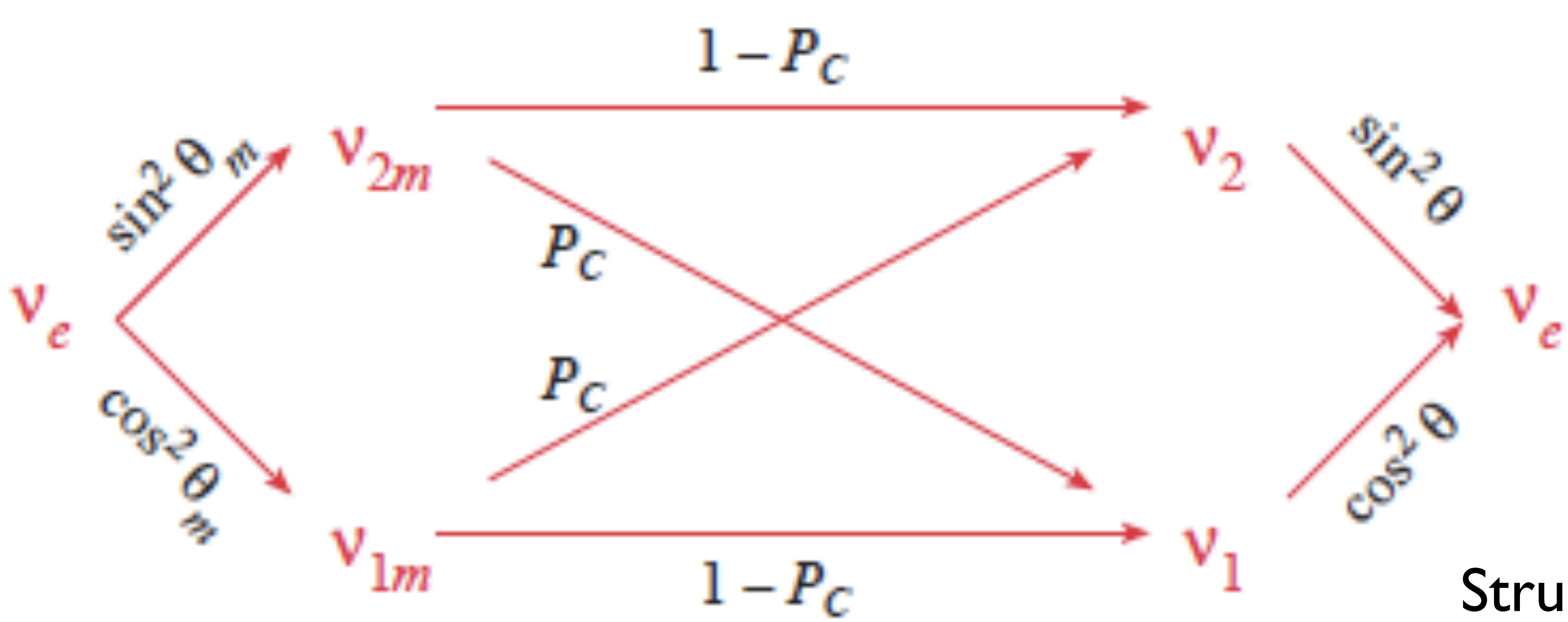
$$\nu_A \quad \text{is} \quad \cos^2\theta_m$$
$$\nu_B \quad \text{is} \quad \sin^2\theta_m$$

If matter effects dominate, $\sin^2\theta_m \simeq 1$

- propagation as a instantaneous state with many oscillations as the wavelength is much smaller than the propagation distance

$$\nu_B(r \sim 0) \rightarrow \begin{array}{ll} \nu_B(\text{surface}) = \nu_2 & \text{with probability } 1 - P_C \\ \nu_A(\text{surface}) = \nu_1 & \text{with probability } P_C \end{array}$$

- in presence of adiabaticity, no jumps
- finally, neutrinos propagate from the Sun to the Earth



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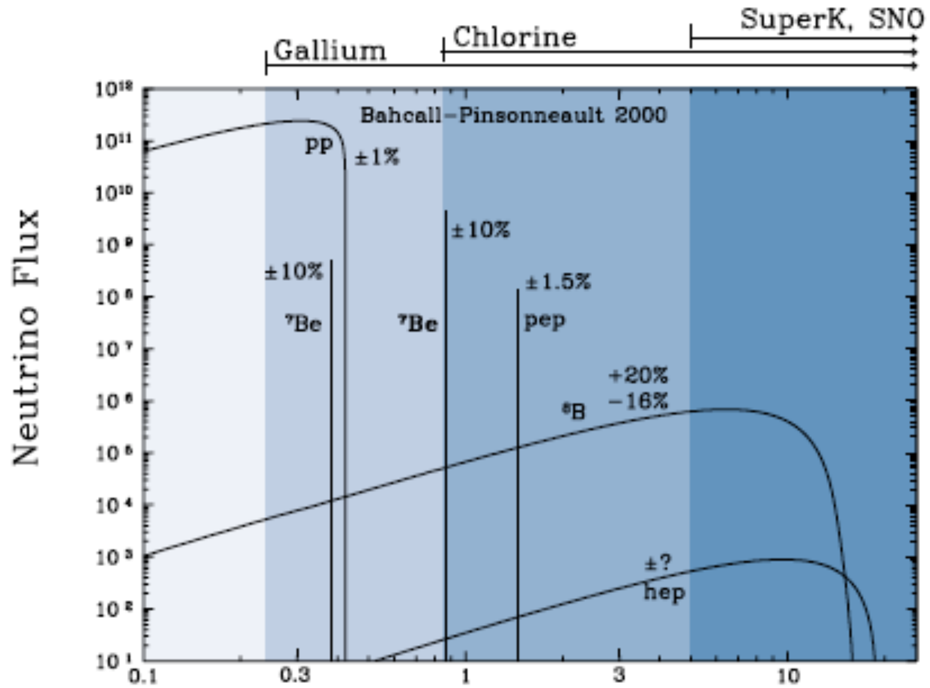
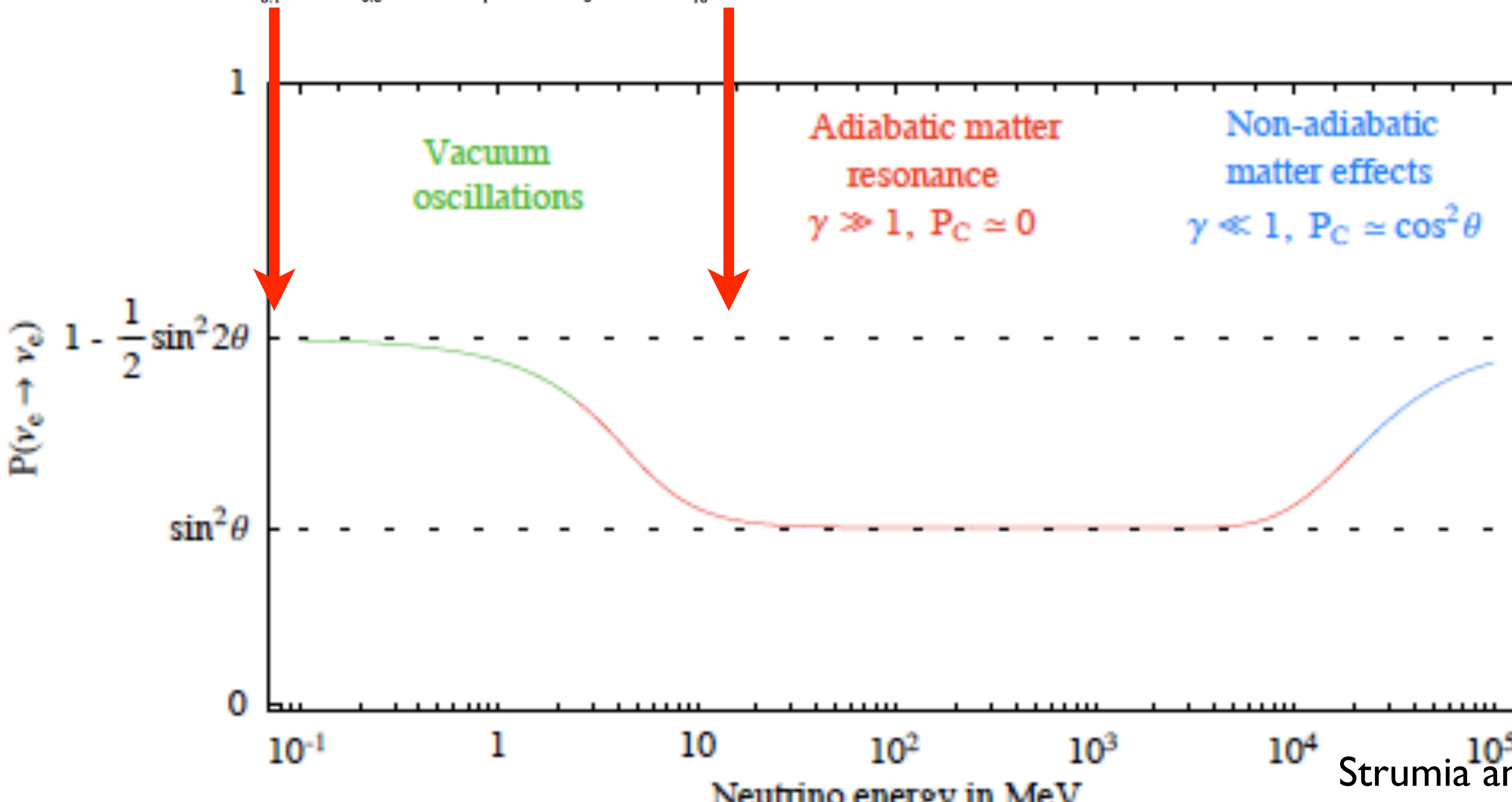
The survival probability is finally given by

$$P(\nu_e \to \nu_e) = \frac{1}{2} + (\frac{1}{2} - P_C)\cos(2\theta)\cos(2\theta_m(r \sim 0))$$

- $P(\nu_e \to \nu_e) = 1 - \frac{1}{2}\sin^2(2\theta)$ (averaged vacuum oscillations), when matter effects are negligible (low energies)
- $P(\nu_e \to \nu_e) = \sin^2\theta$ (dominant matter effects and adiabaticity) (high energies)



Solar neutrinos have energies which go from vacuum oscillations to adiabatic resonance.



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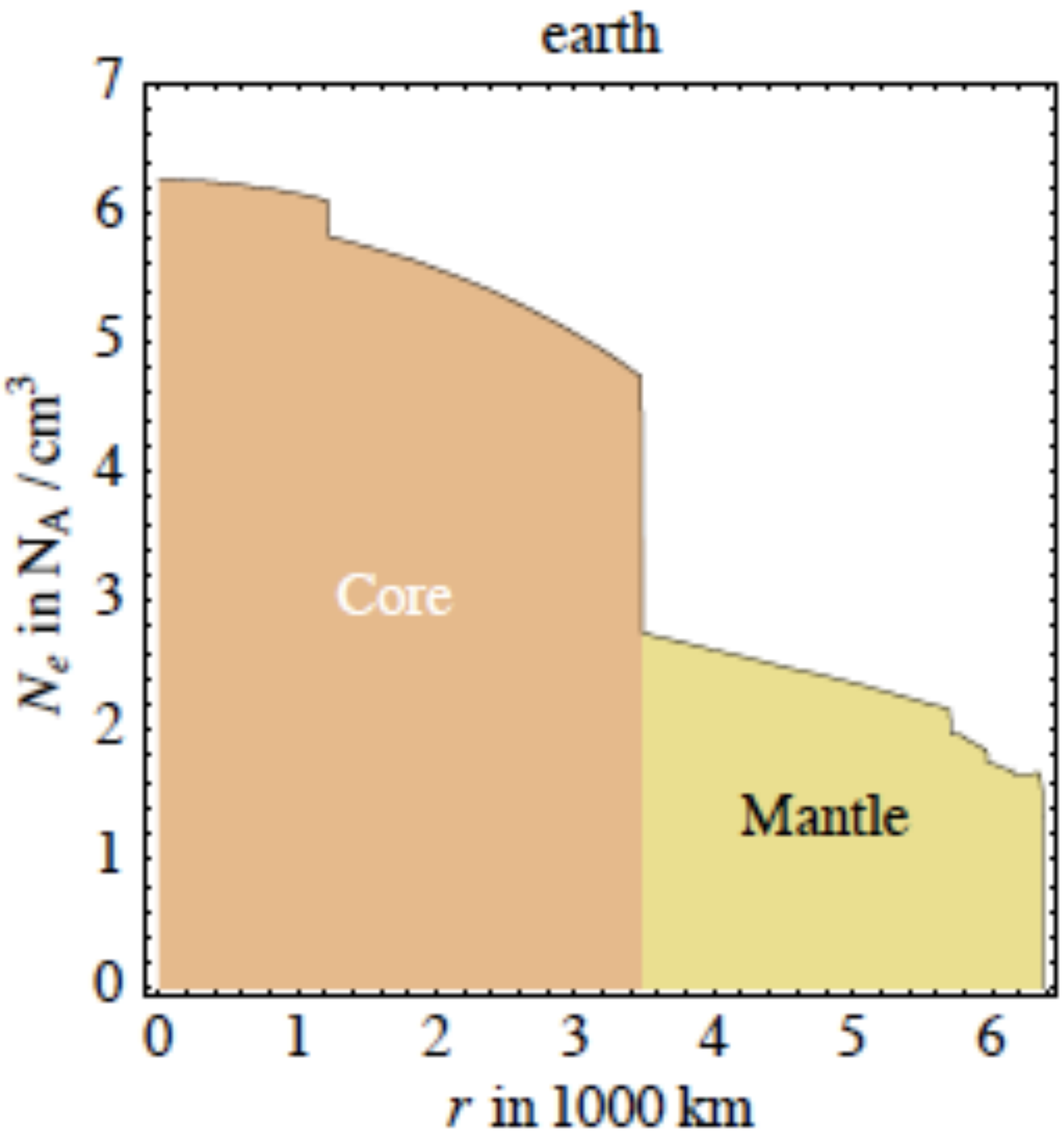
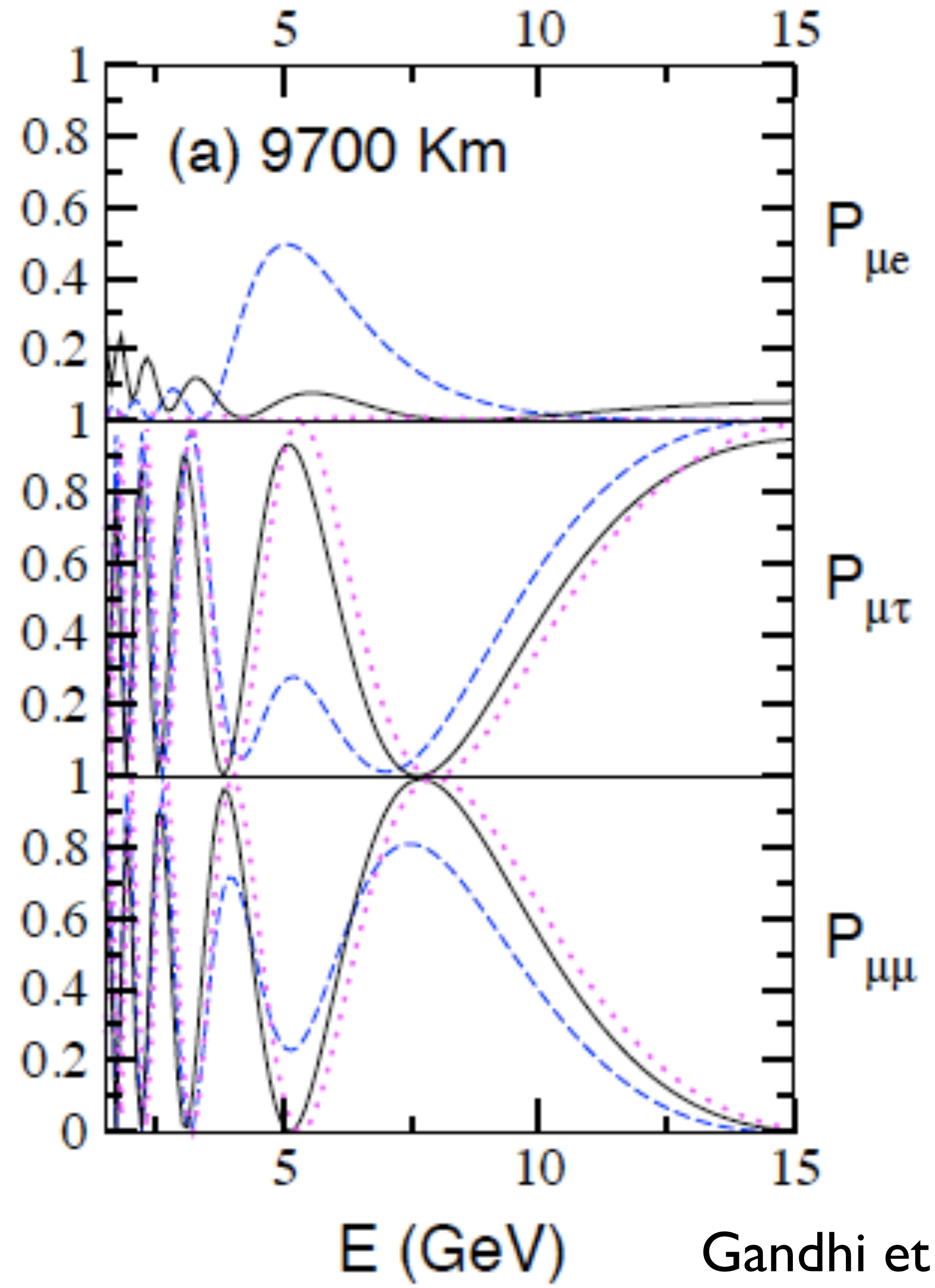
Oscillation is Earth

In the Earth the profile can be well approximated by two constant density regions: the mantle and the core.



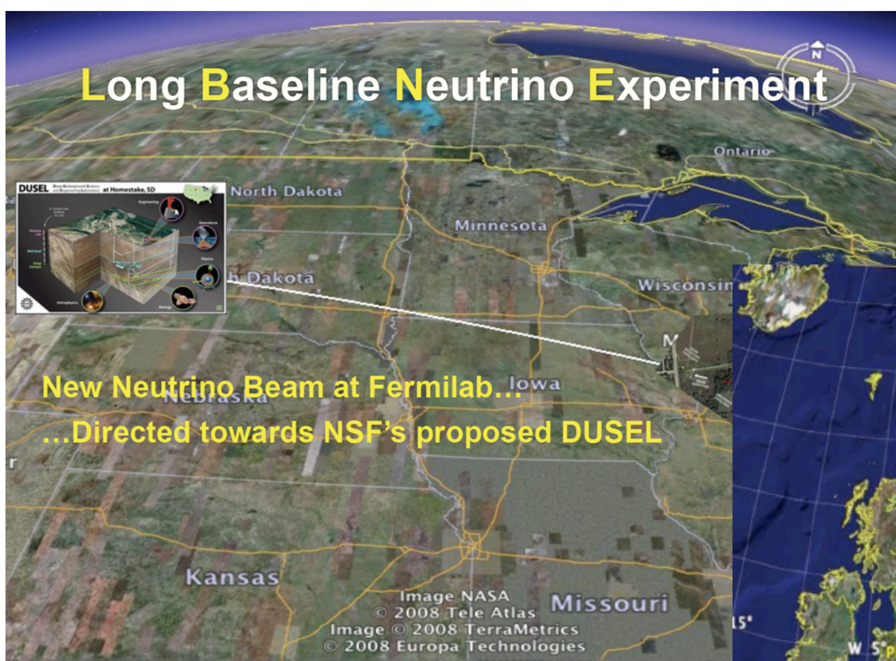


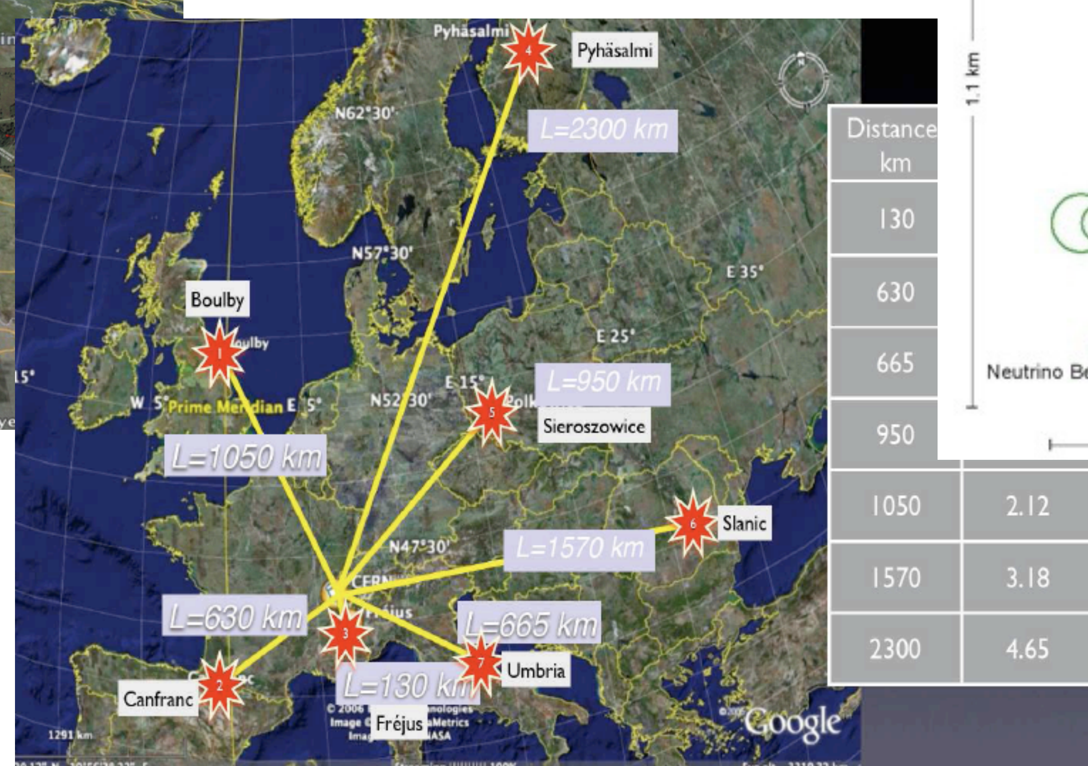
Gandhi et al., 2004

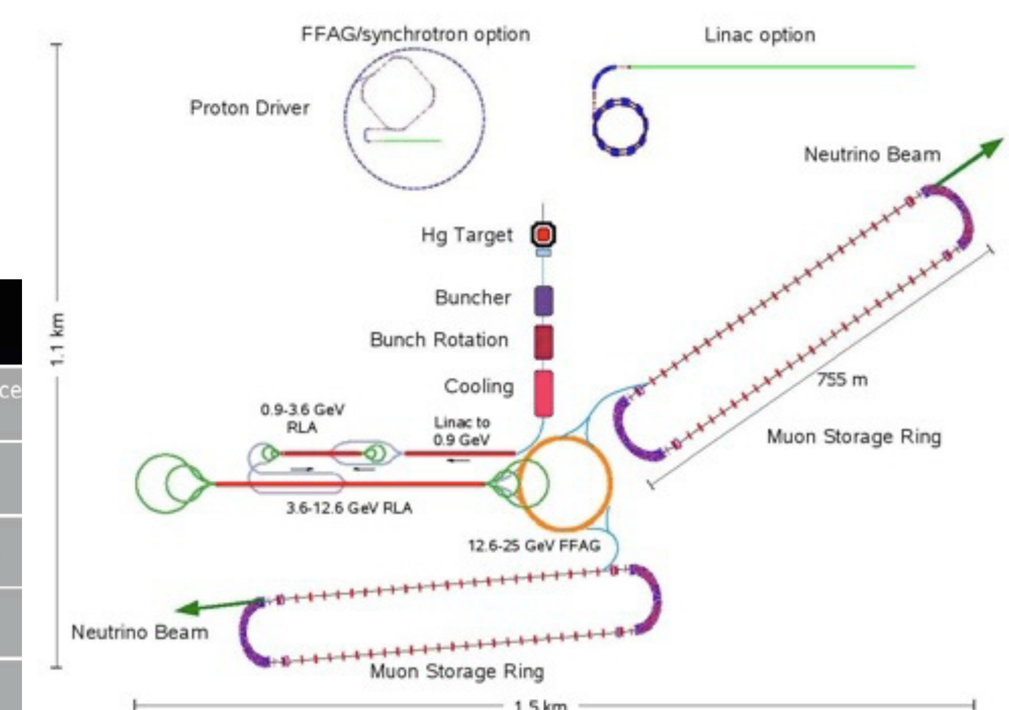
3-neutrino oscillations in the crust

There are long-baseline neutrino experiments which look for oscillations $\nu_\mu \Rightarrow \nu_e$ both for CPV and matter effects.

For distances, 100-3000 km, we can assume that the Earth has constant density, but we need to take into account 3-neutrino mixing effects.







One can compute the probability by expanding the full 3-neutrino oscillation probability (see previous lecture) in the small parameters $\theta_{13}, \Delta m^2_{\rm sol}/\Delta m^2_A$.

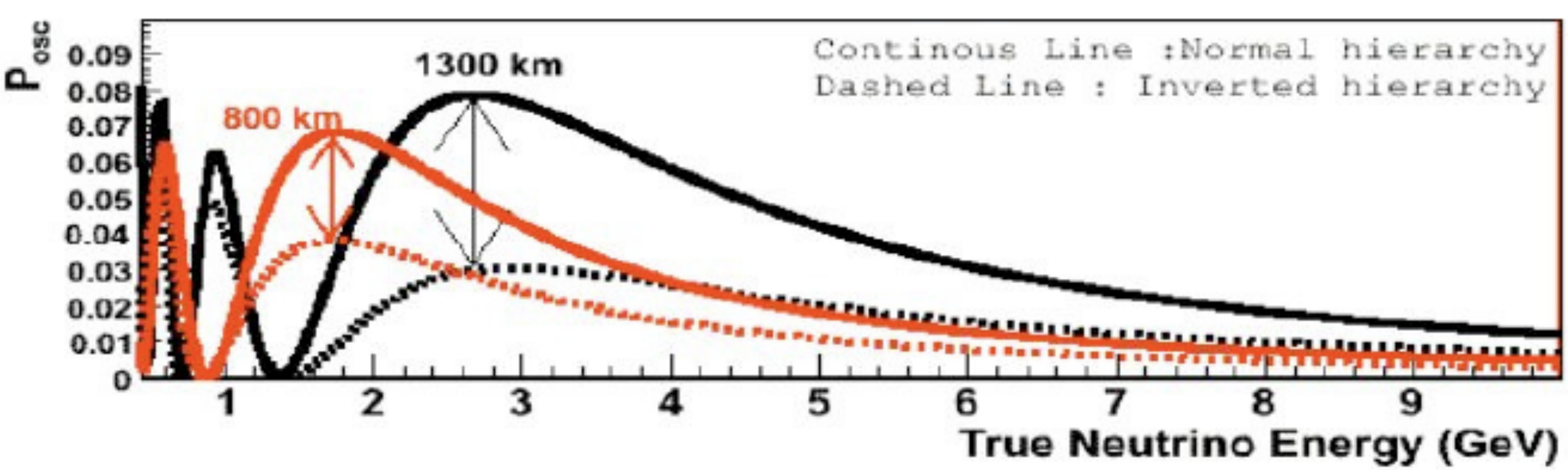
$$P(\bar{P}) \simeq s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{A\mp\Delta_{13}}\right)^2 \sin^2 \frac{(A\mp\Delta_{13})L}{2}$$

$$+\tilde{J}\frac{\Delta_{12}}{A}\frac{\Delta_{13}}{A\mp\Delta_{13}} \sin\frac{AL}{2} \sin\frac{(A\mp\Delta_{13})L}{2} \cos\left(\mp\delta + \frac{\Delta_{13}L}{2}\right)$$

$$+c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\frac{AL}{2}$$

Matter effects

CP violation



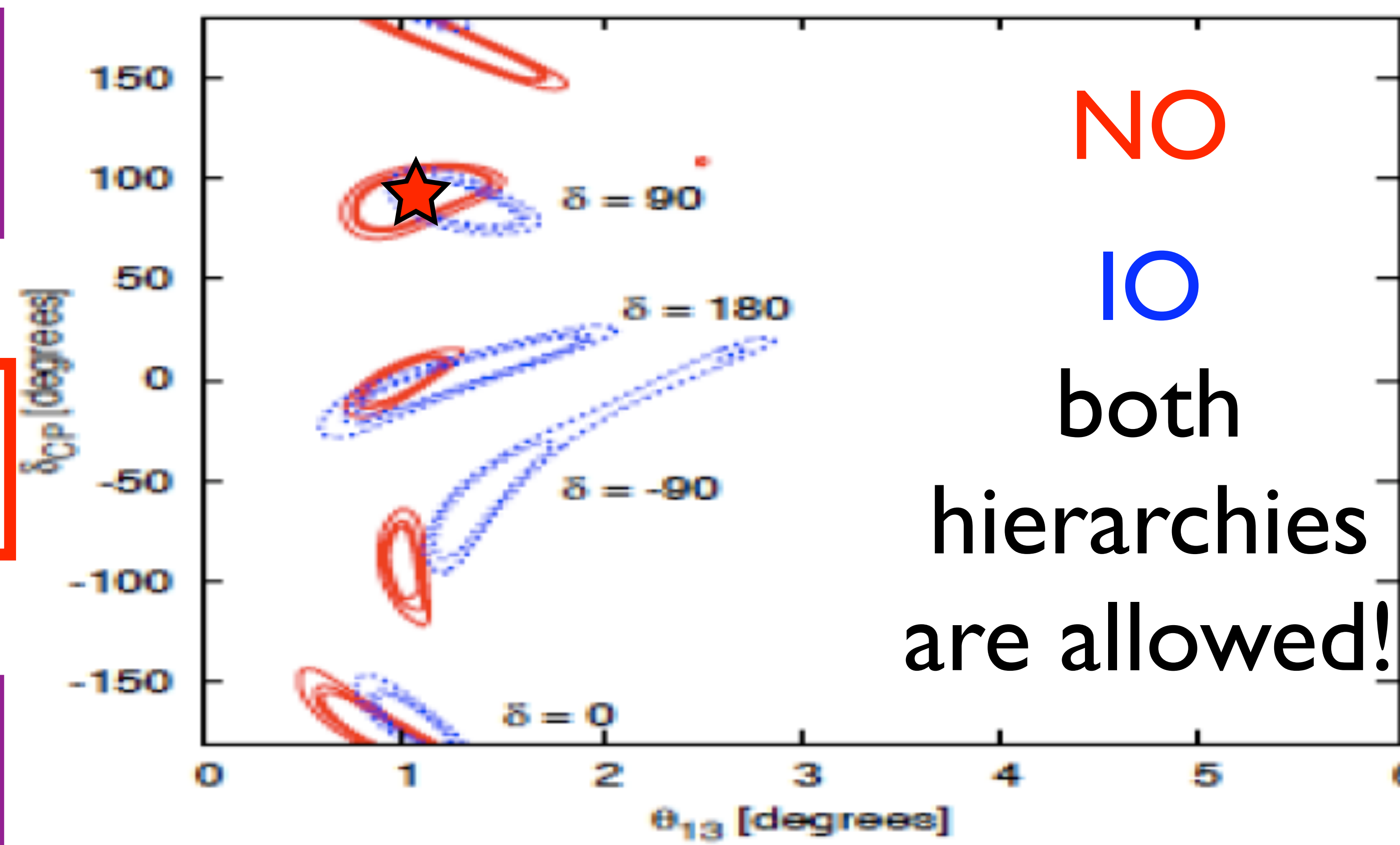
Degeneracies

The determination of CPV and the mass ordering is complicated by the issue of **degeneracies**: different sets of parameters which provide an equally good fit to the data (eight-fold degeneracies).

$$\theta_{13}, \delta, \mathrm{sgn}(\Delta m^2_{31}), \theta_{23}$$

$$P(L/E) \quad \text{and} \quad \bar{P}(L/E)$$

$$\theta'_{13}, \delta', \mathrm{sgn}'(\Delta m^2_{31}), \theta'_{23}$$



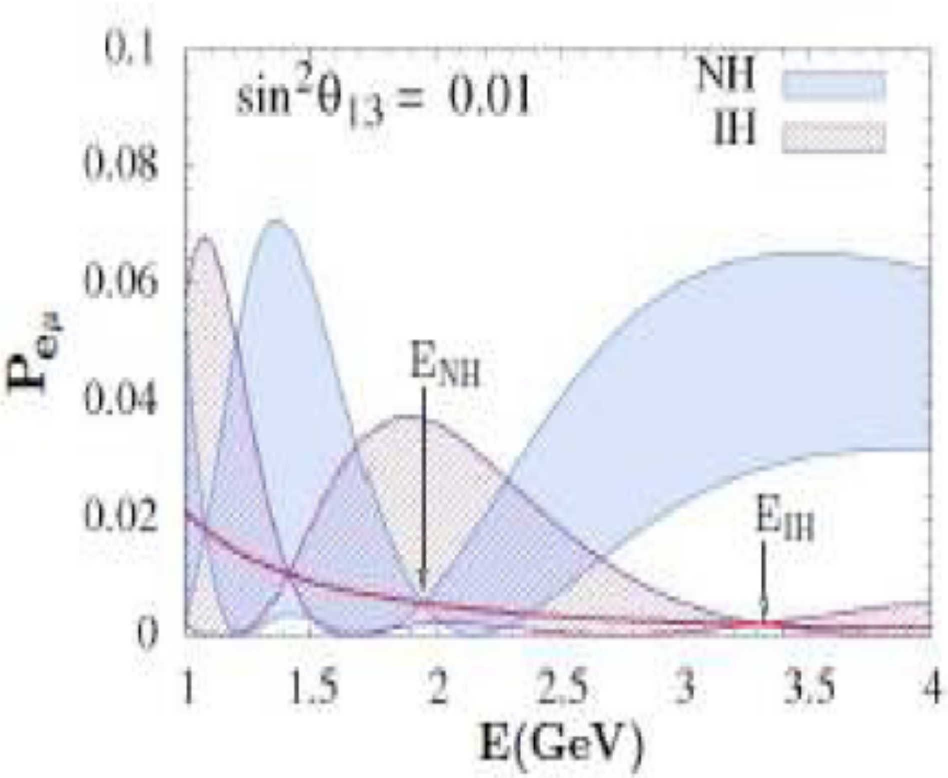
- (θ_{13}, δ) degeneracy (Koike, Ota, Sato; Burguet-Castell et al.)

$$\delta' = \pi - \delta$$
$$\theta'_{13} = \theta_{13} + \cos\delta \sin 2\theta_{12} \frac{\Delta m^2_{12} L}{4E} \cot\theta_{23} \cot \frac{\Delta m^2_{13} L}{4E}$$

Having information at different L/E can resolve this.

- sign(Δm^2_{31}) vs CPV (matter effects). In vacuum:

$$\delta' \to \pi - \delta \qquad \mathrm{sign}'(\Delta m^2_{13}) \to -\mathrm{sign}(\Delta m^2_{13})$$



This degeneracy is broken by matter effects.

For ex. Bimagic baseline at L=2540 km
Excellent sensitivity to the hierarchy
A. Dighe et al., 1009.1093; Raut et al. 0908.3741; Joglekar et al. 1011.1146

- the octant of θ_{23} (low E data) (Fogli, Lisi)

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- **Neutrino oscillations in experiments: linking the theory to experimental situations**

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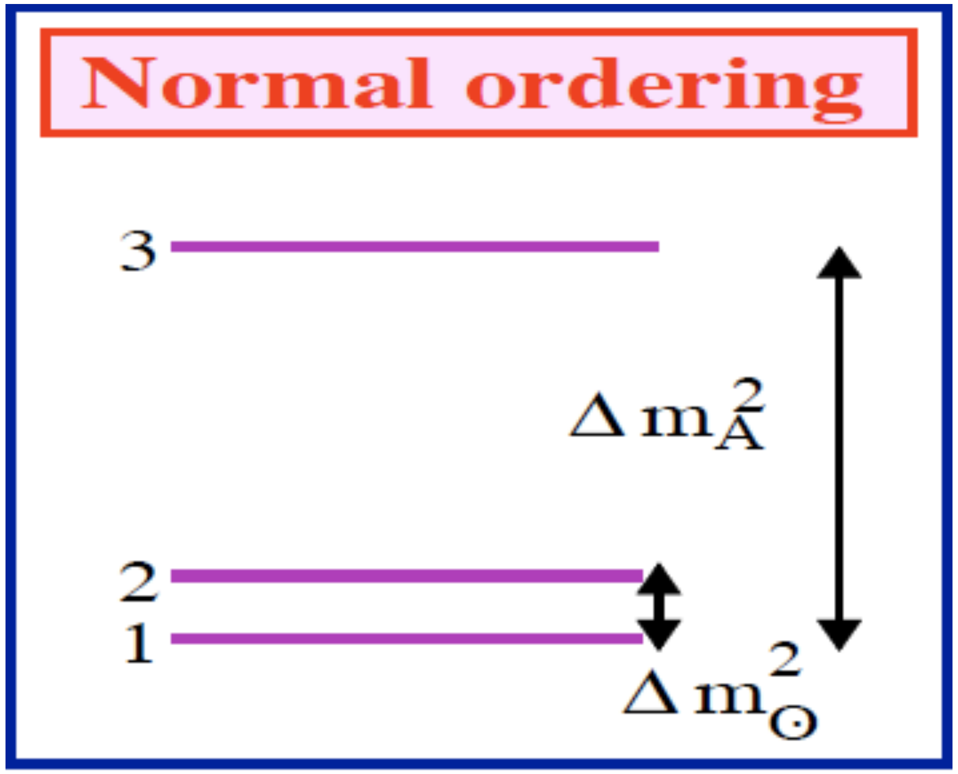
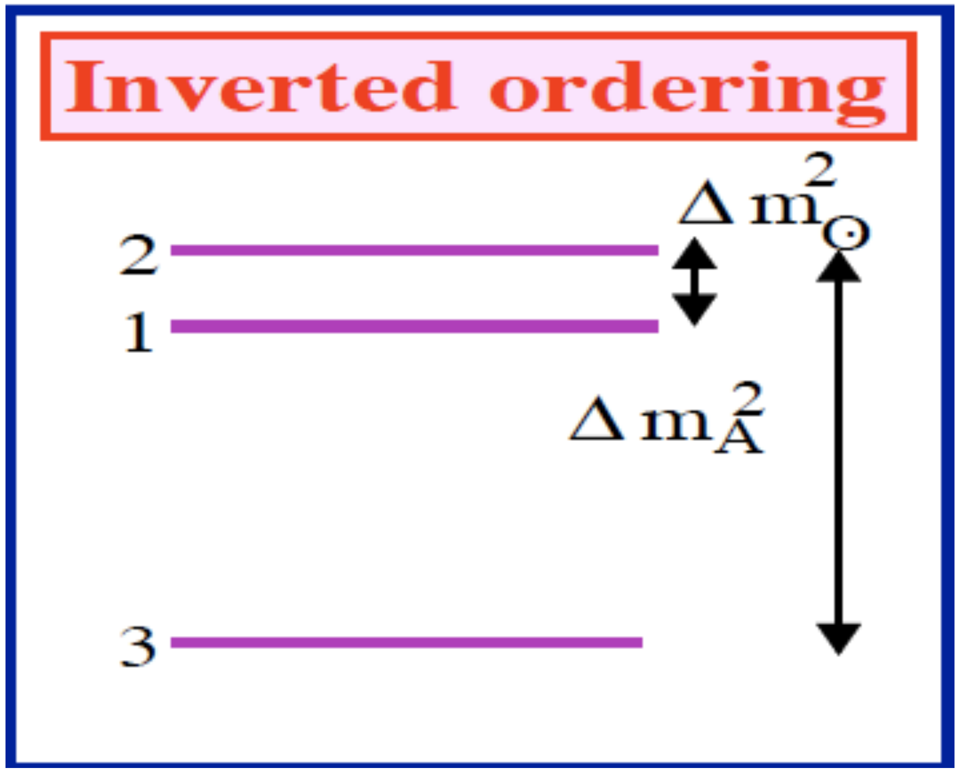
Implications of neutrino oscillations

$\Delta m_s^2 \ll \Delta m_A^2$ implies at least 3 massive neutrinos.





$$m_1 = m_{\rm min}$$
$$m_2 = \sqrt{m_{\rm min}^2 + \Delta m_{\rm sol}^2}$$
$$m_3 = \sqrt{m_{\rm min}^2 + \Delta m_{\rm A}^2}$$

$$m_3 = m_{\rm min}$$
$$m_1 = \sqrt{m_{\rm min}^2 + \Delta m_{\rm A}^2 - \Delta m_{\rm sol}^2}$$
$$m_2 = \sqrt{m_{\rm min}^2 + \Delta m_{\rm A}^2}$$

Measuring the masses requires: $m_{\rm min}$ and the ordering.

Mixing is described by a unitary mixing matrix.

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar, reactor $\theta_\odot \sim 30^o$ — Atm, Acc. $\theta_A \sim 45^o$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{-i\alpha_{31}/2+i\delta} \end{pmatrix}$$

CPV phase — Reactor, Acc. $\theta < 12^o$ — CPV Majorana phases

As discussed yesterday, there is 1 "Dirac" CP-violating phase (entering neutrino oscillations), and 2 Majorana CP-violating phases entering lepton number violating processes, such as neutrinoless double beta decay.

Phenomenology questions for the future

- What is the nature of neutrinos (Majorana vs Dirac)?

- What are the values of the masses?

Direct mass searches + Cosmology

Neutrinoless double beta decay

- Is there CP-violation?

LBL

- What are the precise values of mixing angles (tribimaximal mixing?)?

Reactor

- Is the standard picture correct?

Atmospheric, solar

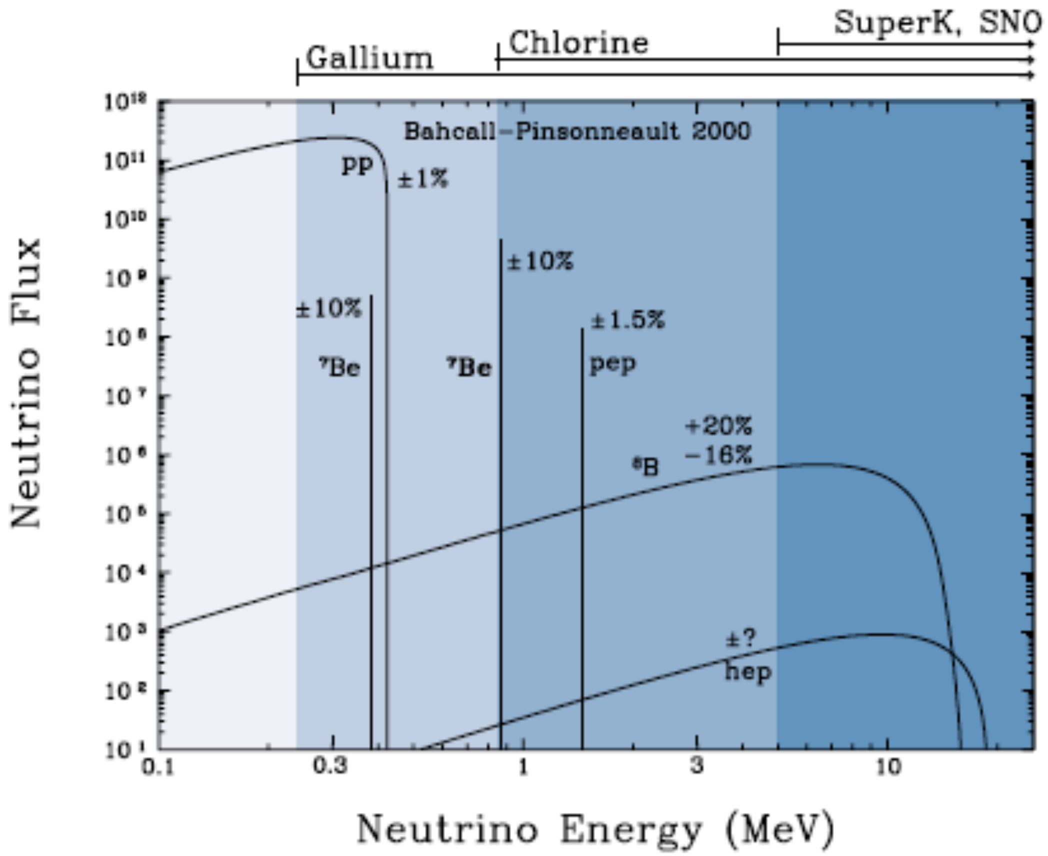
A wide experimental programme is under way or at the proposal stage (DoubleCHOOZ, Daya Bay, RENO, LBL exp, i.e. superbeams, betabeams, neutrino factory) as well as other searches: solar (Borexino), atmospheric (megaton-scale detector, INO), supernova neutrinos, SBL exp for sterile neutrino searches.

Where neutrino oscillations are relevant

linking the theory with experiments (see lectures by Lasserre, Rubbia, Katsanevas)

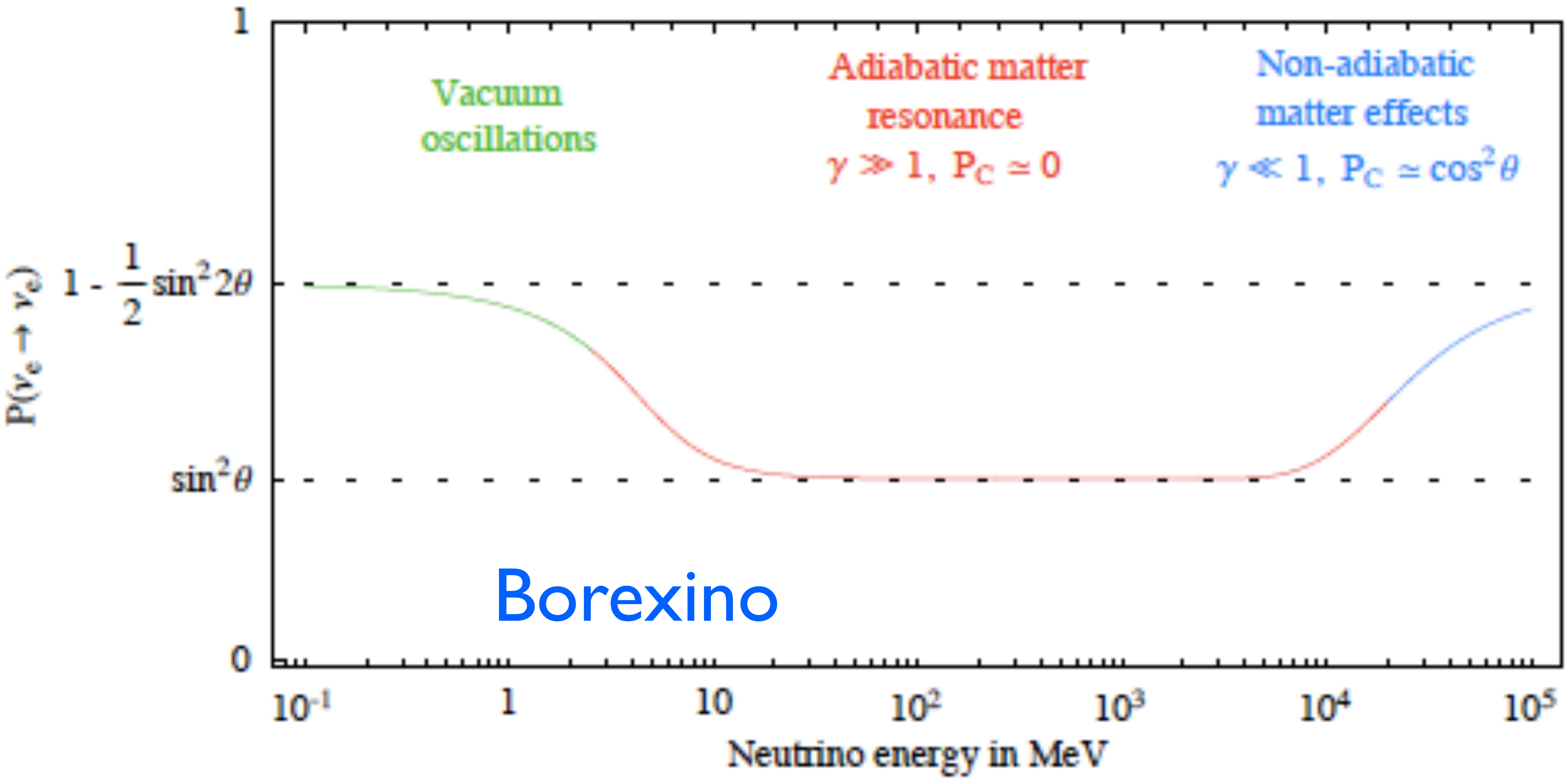
Solar neutrinos

Electron neutrinos are copiously produced in the Sun, at very high electron densities.



http://www.sns.ias.edu/~jnb/

- Typical energies: 0.1-10 MeV.
- MSW effect at high energies, vacuum oscillations at low energy (see previous discussion).
- One can observed CC ν_e and NC: measuring the oscillation disappearance and the overall flux.



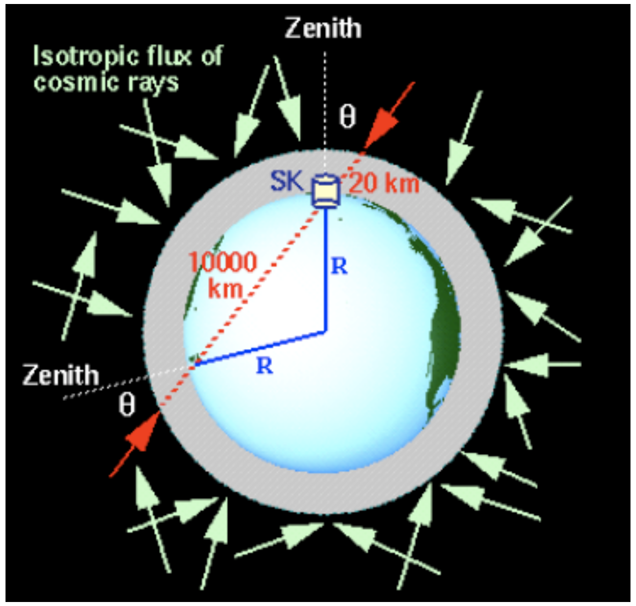
SAGE, GALLEX

SuperKamiokande

SNO

Atmospheric neutrinos.

Cosmic rays hit the atmosphere and produce pions (and kaons) which decay producing lots of muon and electron (anti-) neutrinos.



- Typical energies: 100 MeV - 100 GeV
- Typical distances: 100-10000 km.

- The probabilities of interest are:

$$P(\nu_\mu \to \nu_e; t) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\tau; t) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu; t) = 1 - 4 s_{23}^2 c_{13}^2 (1 - s_{23}^2 c_{13}^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

Reactor neutrinos

Copious amounts of electron antineutrinos are produced from reactors.

- Typical energy: 1-3 MeV;
- Typical distances: 1-100 km.

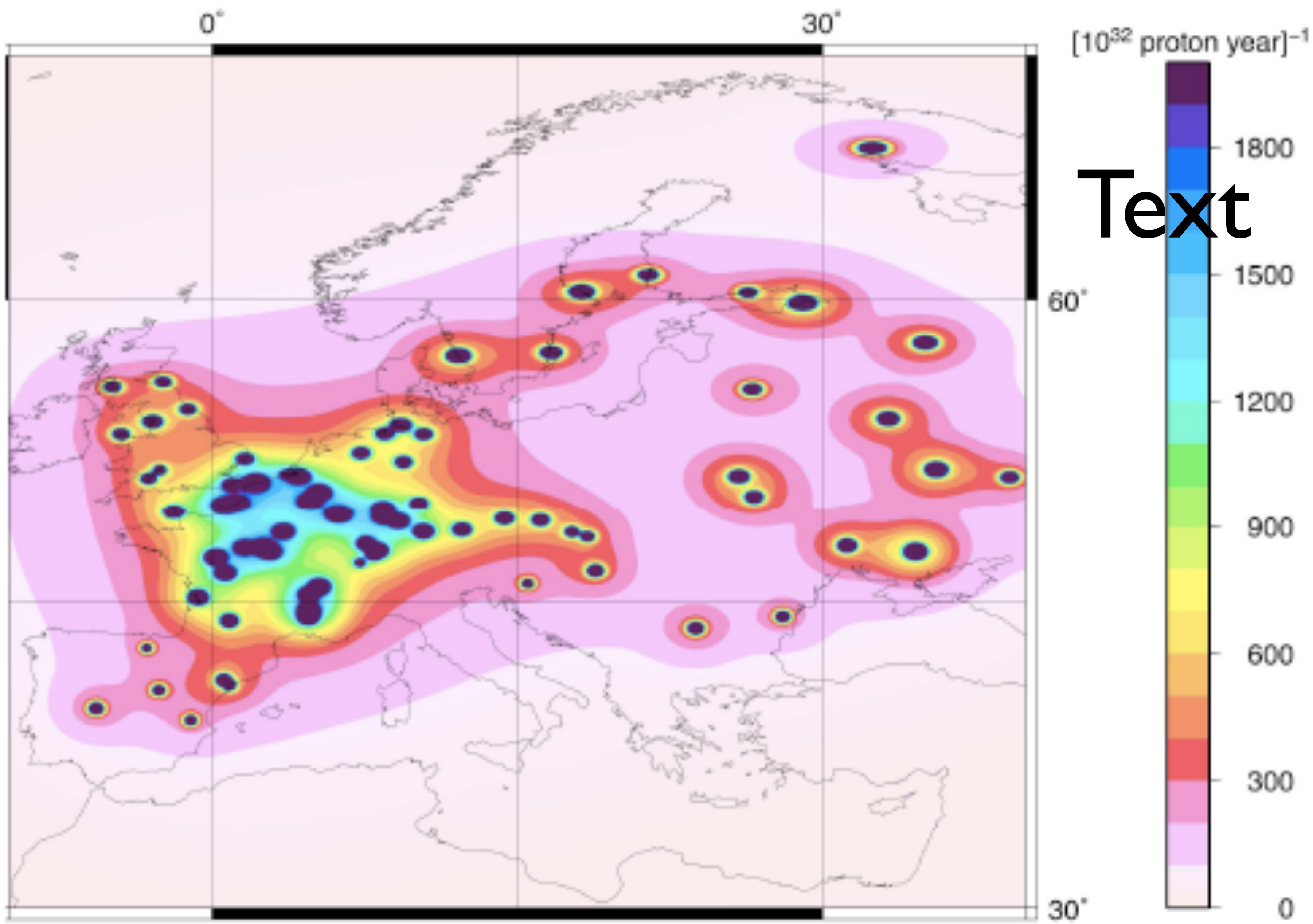
- At these energies inverse beta decay interactions dominate and the disappearance probability is

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; t) \quad = \quad 1 - \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m^2_{31} L}{4E}$$

Sensitivity to θ_{13}. Reactors play an important role in the discovery of θ_{13} and in its precise measurement as they do not suffer from degeneracies from other experiments.

Many new experiments: DoubleCHOOZ, RENO and Daya Bay, searching for θ_{13}.

The diffuse neutrino flux is also a background for other searches (geoneutrinos...).

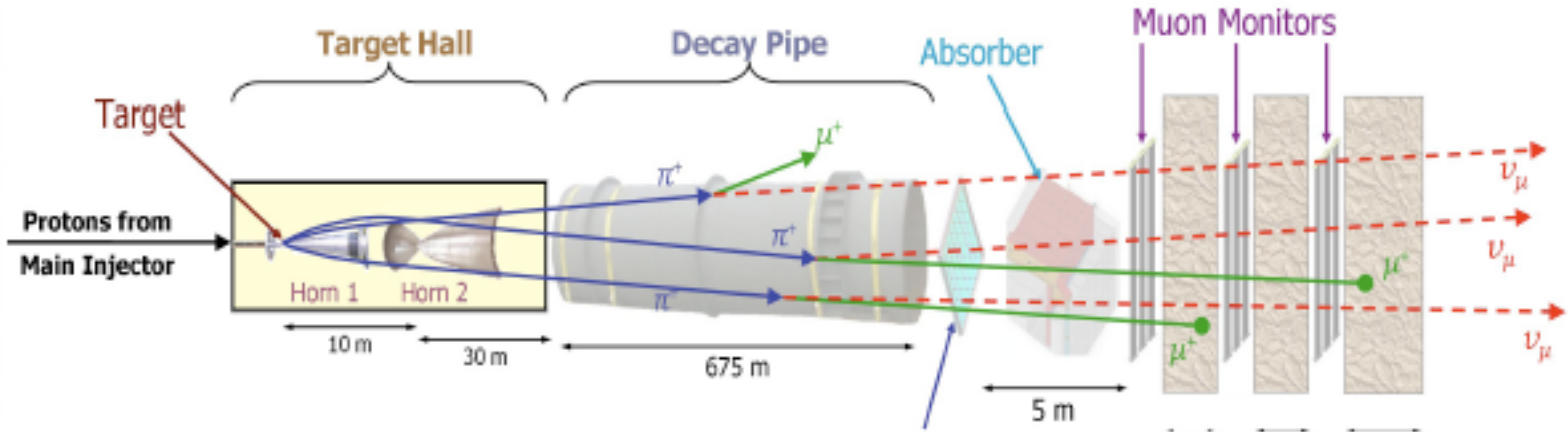


Text

Proposed site	Reactor neutrino background (TNU)
Pyhäsalmi (FIN)	~ 78
Pyhäsalmi after 2012	~ 86
Caso (ITA)	~ 110
Slanic (ROM)	~ 130
Sieroszowice (POL)	~ 200
Canfranc (ESP)	~ 300
Frejus (FRA)	~ 700
Boulby (GBR)	~ 1600
Boulby w/o Hartlepool	~ 280

Accelerator neutrinos

Conventional beams (K2K, MINOS): muon neutrinos produced in pion decays



- Typical energies: 200 MeV - 10 GeV.

MINOS: E~4 GeV; T2K: E~700 MeV; NOvA: E~2 GeV.
OPERA and ICARUS: E~20 GeV.

- Typical distances: 100 km - 2000 km.

MINOS: L=735 km; T2K: L=295 km; NOvA: L=810 km.
OPERA and ICARUS: L=700 km.

At these energies, one can detect electron, muon (and tau) ν via CC interactions.

MINOS: $$P(\nu_\mu \to \nu_\mu; t) = 1 - 4s_{23}^2 c_{13}^2 (1 - s_{23}^2 c_{13}^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

T2K: $$P(\nu_\mu \to \nu_e; t) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

OPERA (and ICARUS): $$P(\nu_\mu \to \nu_\tau; t) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

Sensitivity to $\Delta m_{31}^2, \quad \theta_{23}, \quad \theta_{13}$

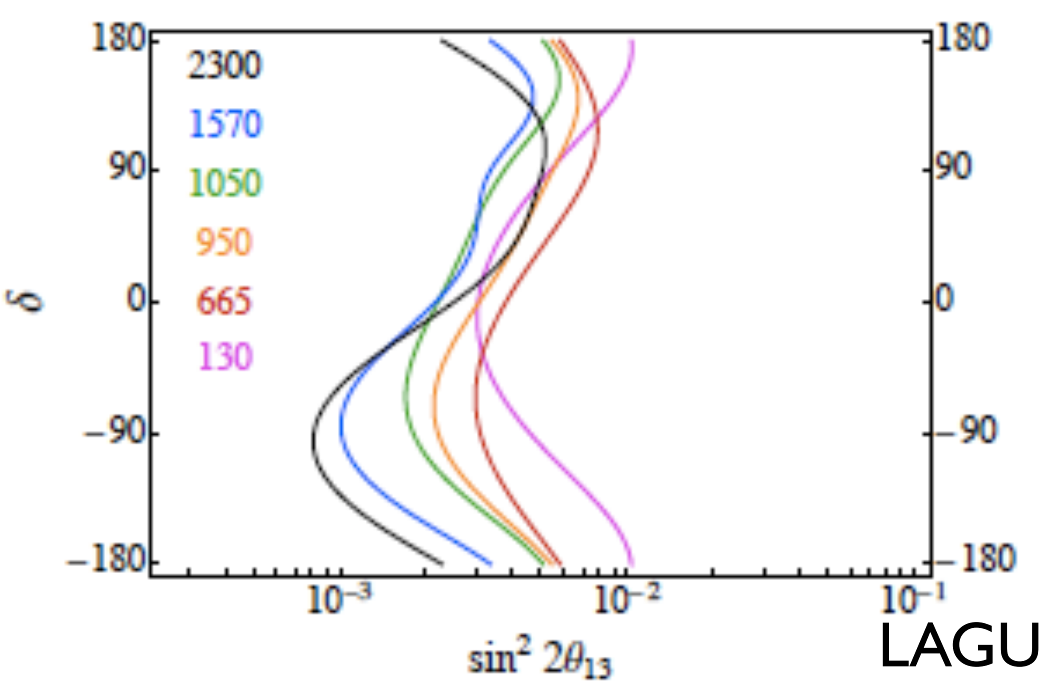
Future long baseline experiments

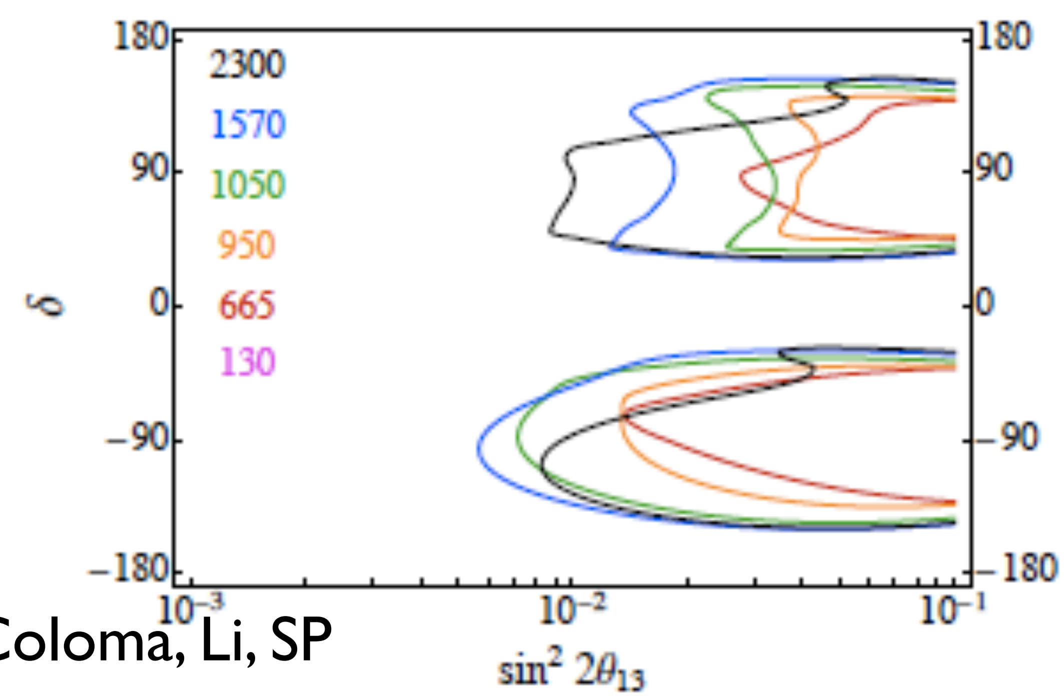
- **superbeams**: upgrade of conventional beams with higher fluxes and larger detectors.

Superbeams have very good reach to CP-violation and matter effects (if L>800 km).

They search for $P_{\mu e}$, $P_{\mu\mu}$ for neutrinos and antineutrinos.

- LBNE: L=1300 km (Fermilab to Homestake), LAGUNA-LBNO (CERN-Frejus or CERN-Phyasalmi), T2K (JPARC-Kamioka, - Okhinoshima, - Korea).



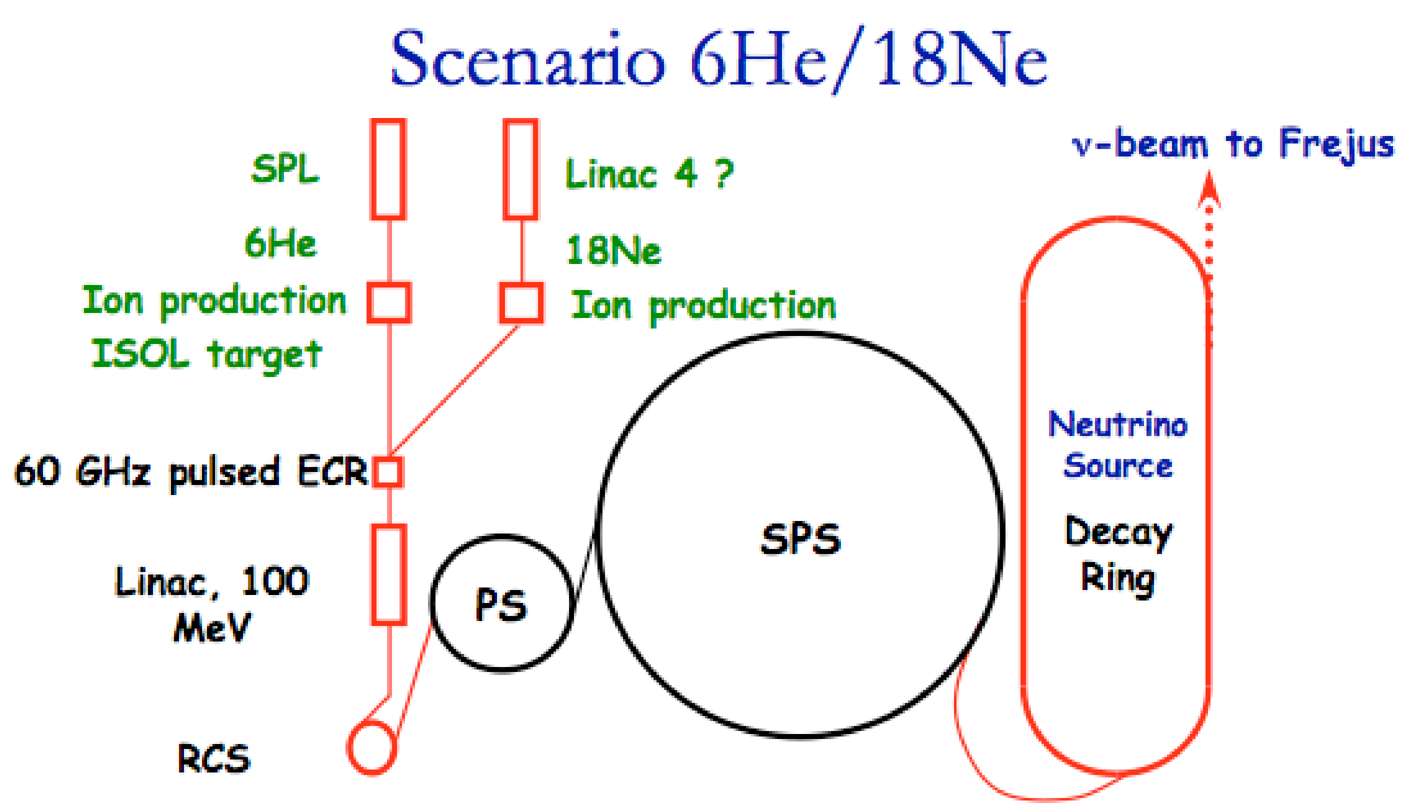


LAGUNA study: Coloma, Li, SP

• **beta beams**: neutrinos are produced in beta decays of highly accelerated ions. The energies are typically low in the currently studied EUROnu betabeam option ($\gamma \sim 100$).

They would search for $P_{e\mu}$ in order to discover CPV. As the typical distance is ~130 km (CERN-Frejus), they have no sensitivity to matter effects.

The combination of betabeam ($P_{e\mu}$) and superbeam ($P_{\mu e}$) is very powerful for CPV as it looks at the T-conjugated channels.



- **neutrino factory**: neutrinos are produced in high gamma muon decays.

right-sign signal

$$\mu^- \to e^- \begin{array}{l} \nu_\mu \quad \longrightarrow \quad \nu_\mu \quad \text{produce } \mu^- \text{ in CC interaction} \\ \bar{\nu}_e \quad \longrightarrow \quad \bar{\nu}_\mu \quad \text{produce } \mu^+ \text{ in CC interaction} \end{array}$$

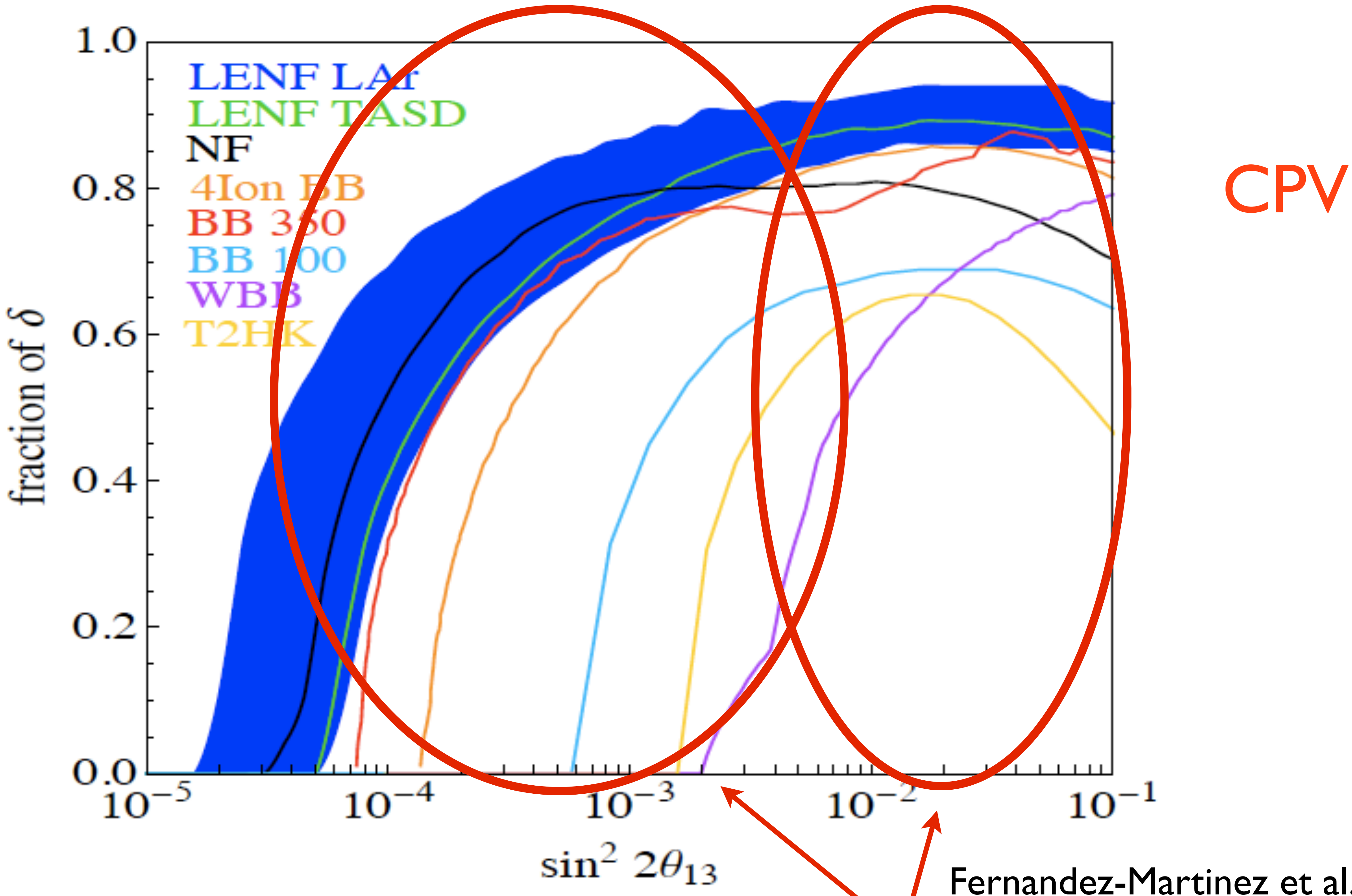
wrong-sign signal

The detector needs to be magnetised in order to distinguish the two signals.

- Right-sign allows to measure $P_{\mu\mu} \Rightarrow \Delta m^2_{31}, \theta_{23}$

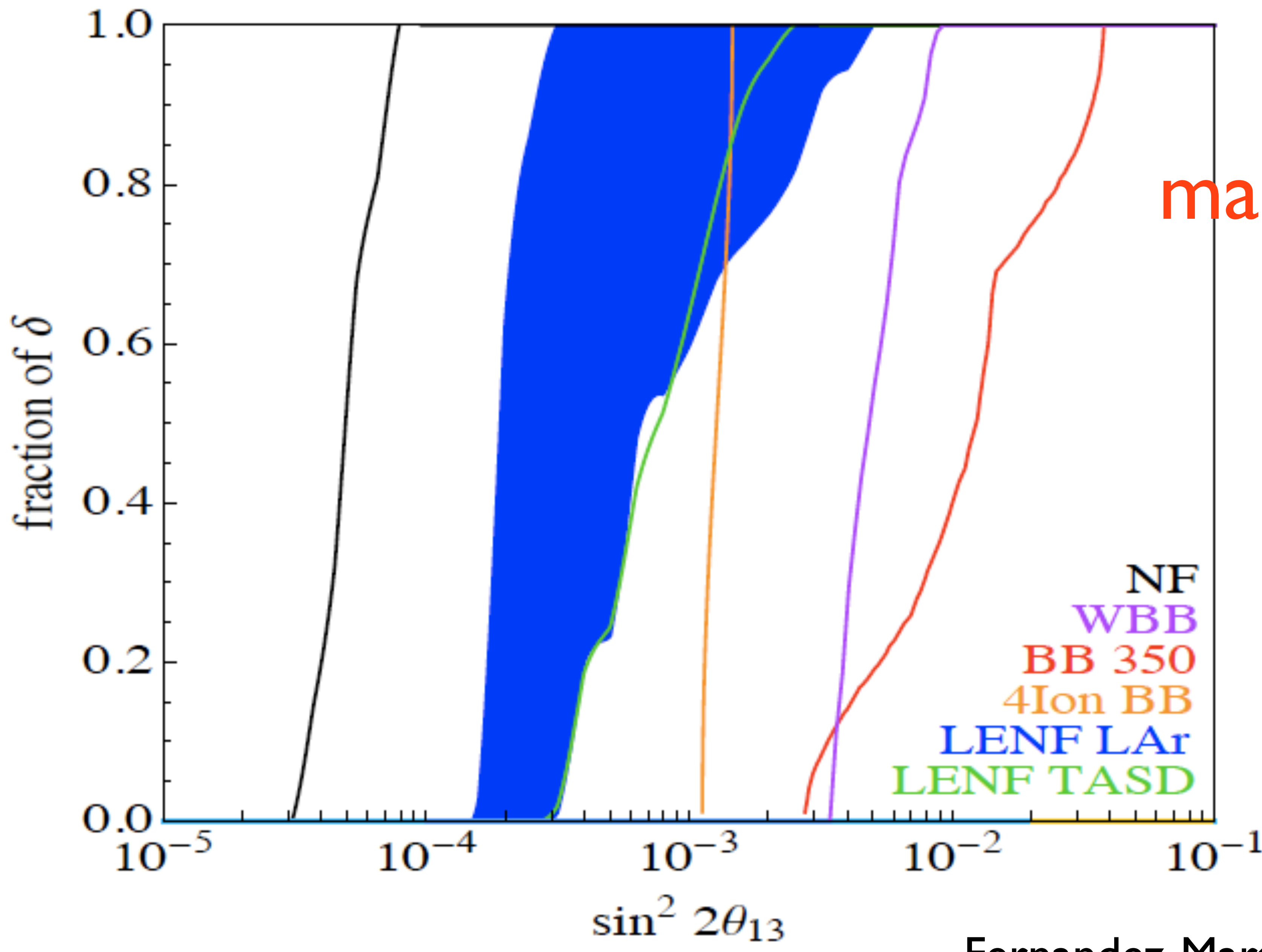
- Wrong-sign signal to measure $P_{e\mu} \Rightarrow \theta_{13}, \delta, \mathrm{sgn}(\Delta m^2_{31})$

High energy neutrino factory (E~ 25 GeV) or low energy one (E~4-10 GeV).



Fernandez-Martinez et al., 0911.3776

Depending on the values of θ_{13} and on the precision required, **different experimental setups and optimisations are required.**



mass ordering

Fernandez-Martinez et al., 0911.3776

Similar considerations hold also for the type of mass ordering, with long baselines (and consequently high energies) preferred for small θ_{13}.

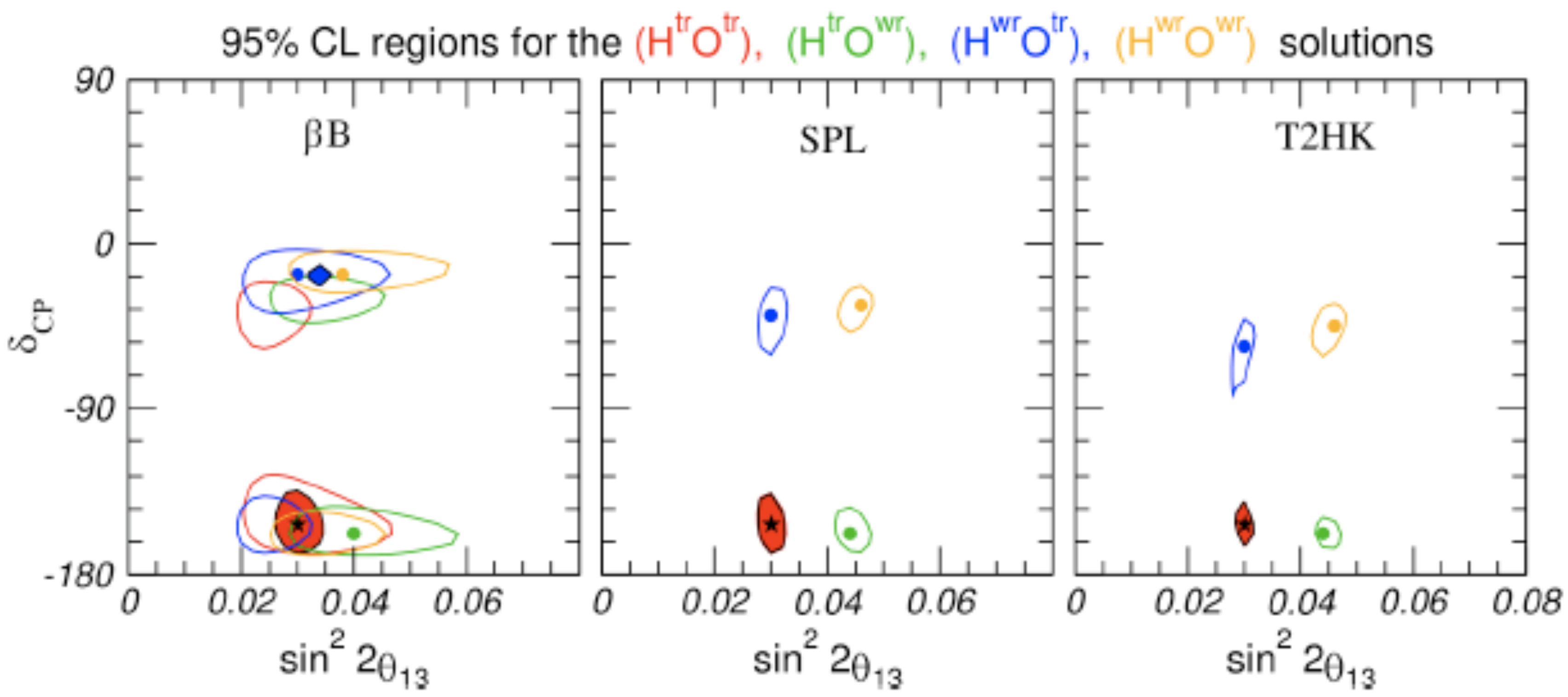
Synergy and complementarity between neutrino oscillation experiments and other searches

- Reactor and LBL experiments:

reactor neutrino measure only θ_{13}. This information can be used to reduce the impact of degeneracies in LBL experiments

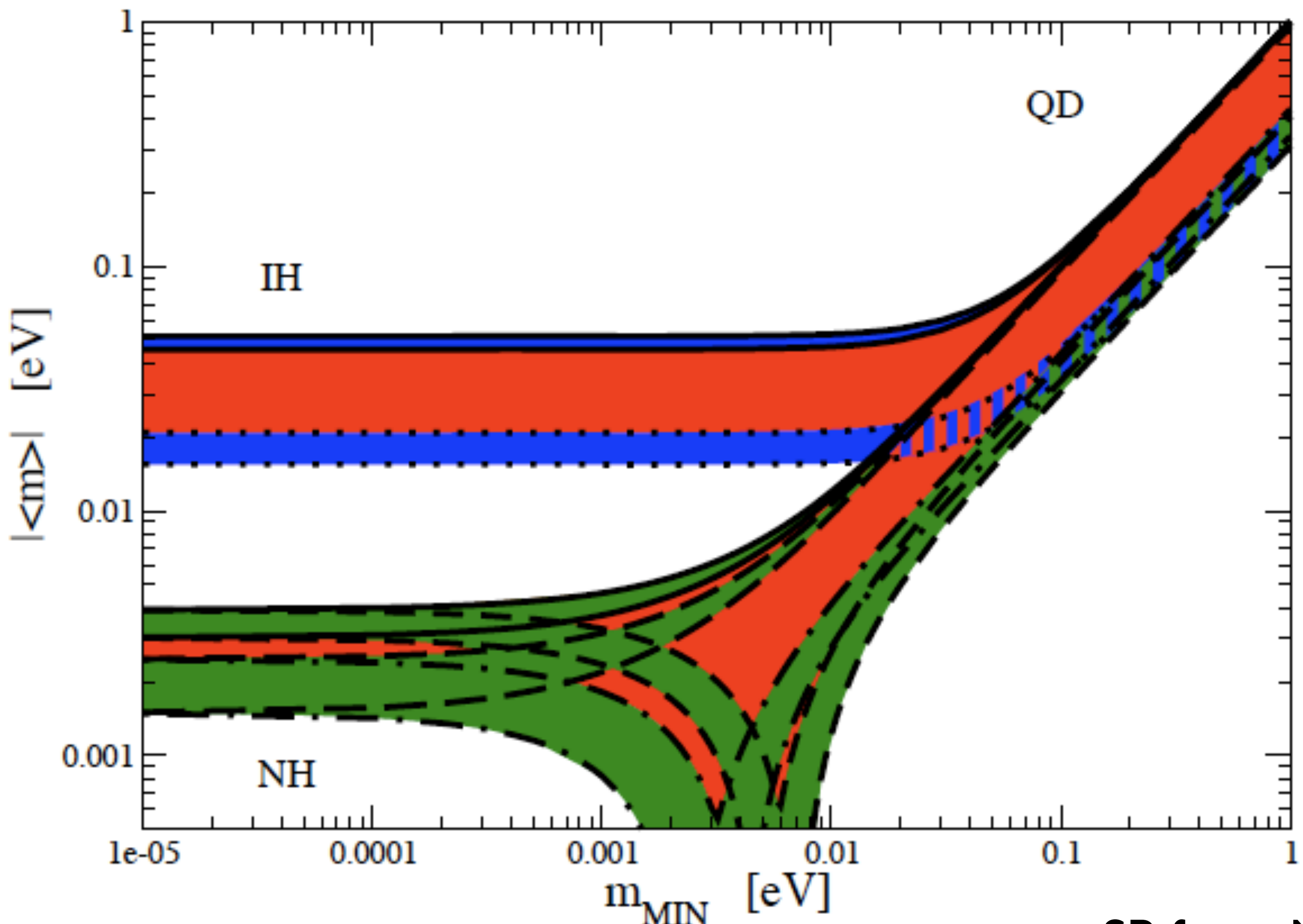
- Atmospheric and LBL experiments:

atmospheric neutrinos are sensitive to matter effects.



Campagne, Maltoni, Mezzetto, Schwetz

Synergy between betabeta-decay and LBL



Klapdor-Kleingrothaus claim 2002 and 2006

Present bounds: Heidelberg-Moscow, IGEX, Cuoricino and NEMO3

Next generation: GERDA, EXO, CUORE, SuperNEMO, SNO+, Majorana, COBRA...

Future experiments: ~1 ton

SP from Nakamura, Petcov review in PDG

If LBL finds NO and neutrinoless double beta decay excludes |<m>|>10 meV, then probably neutrinos are Dirac particles.

- A very wide-experimental neutrino oscillation program is under way or at the proposal stage.

- In the next few years, it will allow to discover θ_{13} and in the coming future to search for CP-violation and the neutrino mass hierarchy, and to measure with precision with mixing angles.

- A strong effort should be performed to control backgrounds and systematic errors.

Plan of lecture II

- Matter effects in neutrino oscillations
  - Matter potential
  - 2-neutrino oscillations in constant density
  - 2-neutrino oscillations in varying density and the MSW effect for solar neutrinos
  - 3-neutrino oscillations in matter for LBL
  - other effects

- Neutrino oscillations in experiments: linking the theory to experimental situations

- **Neutrino oscillations in cosmology: the case of sterile neutrinos**

Neutrino oscillations in cosmology

see also lectures by Hannestad and Volpe

In the early instants of the Universe, neutrinos were in thermal equilibrium in the bath as their interactions were fast enough.

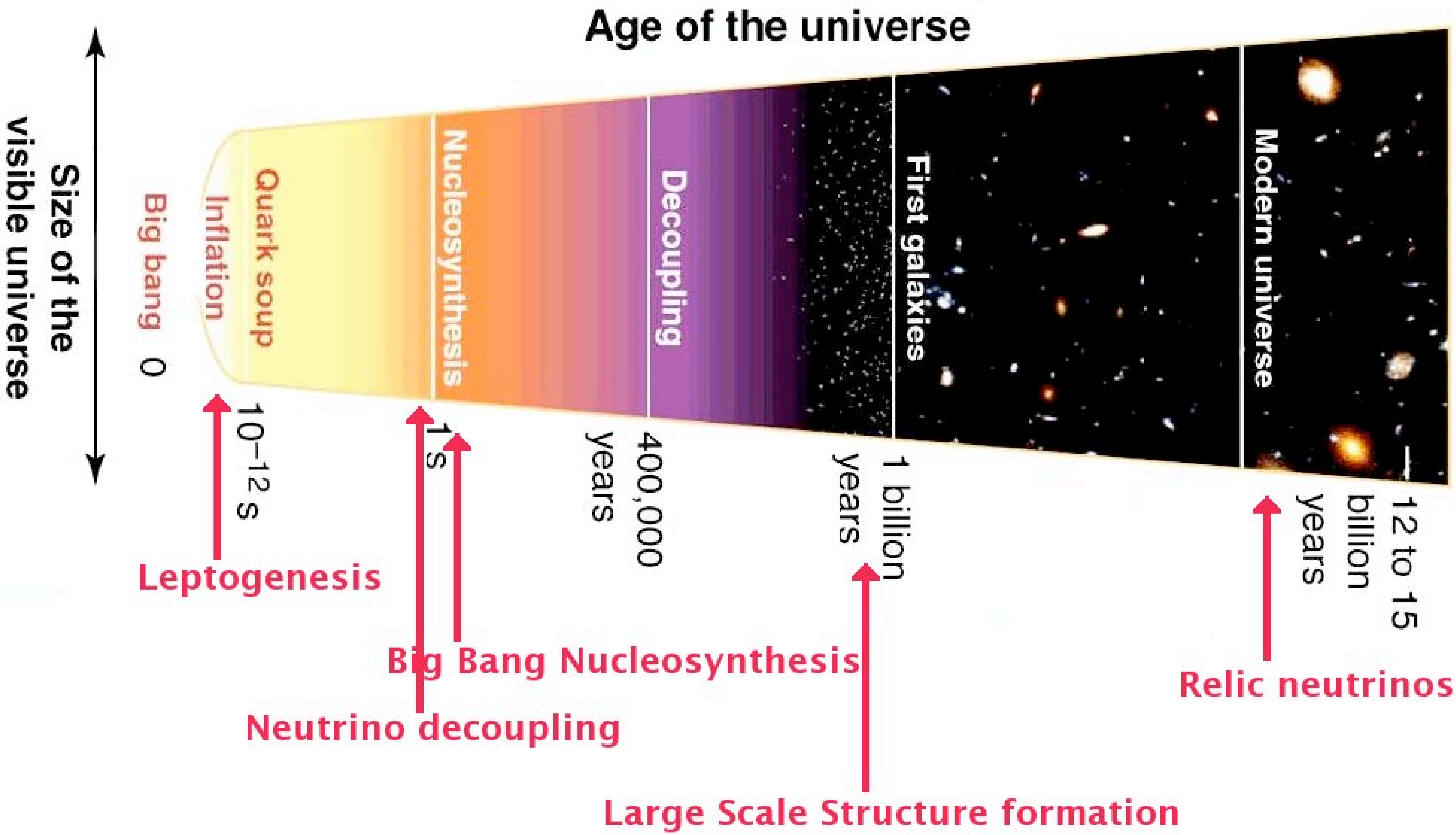
$$\Gamma \sim H$$

$$\Gamma = \langle \sigma \rangle n \sim G_F^2 T^2 \quad T^3$$

$$H = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{m_{\mathrm{Pl}}}$$

Neutrinos get out of equilibrium around MeV and they remain as a background whose momentum redshifts with time.

They are very abundant and can impact on the evolution of the Universe at different stages.



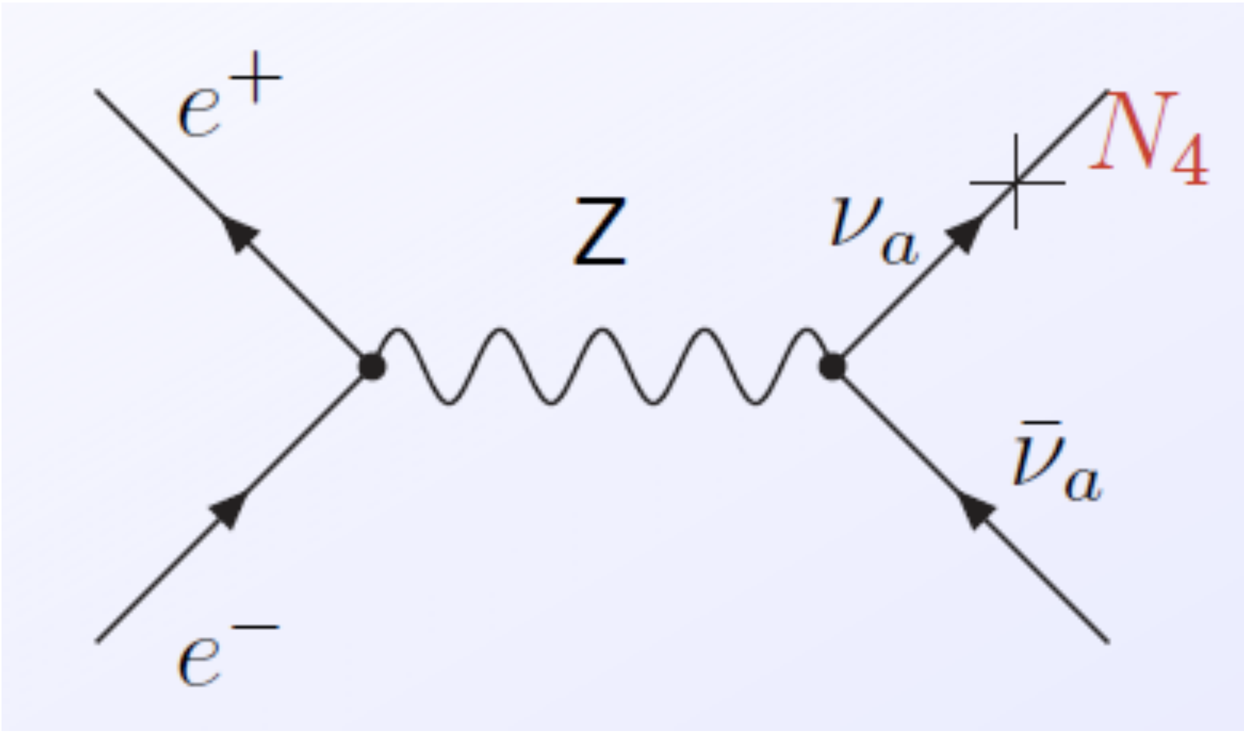
- In cosmology neutrino oscillations are usually not a dominant effect, except for some significant contribution to BBN. (see Hannestad's talk)

- However, oscillations into sterile neutrinos could be important and can be the dominant mechanism of sterile neutrino production. Sterile neutrinos were not in thermal equilibrium if mixing is small and they do not have significant interactions.

- The mixing is given by

$$\begin{pmatrix} \nu_1 \\ N \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_\alpha \\ \nu_s \end{pmatrix}$$

In an interaction involving active neutrinos, an N can be produced due to loss of coherence.



Dodelson, Widrow, 1992

The nearly-sterile neutrino N production

- depends on $\sin^2\theta$
- happens till there are active neutrino interaction, is controlled by Γ and stops at decoupling.

The probability of N production can be computed by looking at the evolution of states. In the flavour basis:

$$i\frac{d}{dt}\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_s\rangle \end{pmatrix} = \left[U\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}U^\dagger + \begin{pmatrix} V_\alpha & 0 \\ 0 & 0 \end{pmatrix}\right]\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_s\rangle \end{pmatrix}$$

One needs to diagonalise H using a unitary transformation

$$U_m(\theta_m) = \begin{pmatrix} \cos\theta_m & -\sin\theta_m \\ \sin\theta_m & \cos\theta_m \end{pmatrix}$$

and finally the average probability of oscillation is given by

$$\langle P(\nu_\alpha \to \nu_s; p, t)\rangle = \frac{1}{2}\langle \sin^2\theta_m\rangle$$

The mixing angle in the Early Universe depends on

- matter effects due to an asymmetry in the background

$$V_D \sim \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 (\mathcal{L} \pm \eta/4)$$

with $\mathcal{L} = (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/\nu_\gamma$

- finite temperature effects

$$|V_T| = -C_a G_F^2 T^4 E/\alpha$$

We have, with $\Delta(p) \equiv m_N^2/(2E)$

$$\sin^2(2\theta_m) = \frac{\Delta^2(p)\sin^2(2\theta)}{\Delta^2(p)\sin^2(2\theta) + D^2 + (\Delta(p)\cos(2\theta) - V_D + |V_T|)^2}$$

The production of sterile neutrinos can be computed by solving the Boltzmann equation

$$\frac{\partial f_N}{\partial T}\Big|_{E/T} = -\frac{\Gamma_\alpha}{2HT}\sin^2(2\theta_m)[f_\alpha(p,T) - f_s(p,T)]$$

For non-resonant production

- at high T, production is suppressed by the mixing angle
- at small T, by Γ
- the maximum of production happens at

$$T \sim 133 \text{ MeV}(m_N/1 \text{ keV})^{1/3}$$

The final abundance is $\Omega_N h^2 \sim 0.3 \frac{\sin^2(2\theta)}{10^{-8}} \left(\frac{m_N}{10 \text{ keV}}\right)^2$

In presence of a large lepton asymmetry, $\mathcal{L} \equiv (n_\nu - n_{\bar{\nu}})/n_\gamma$, matter effects become important and the mixing angle can be resonantly enhanced.

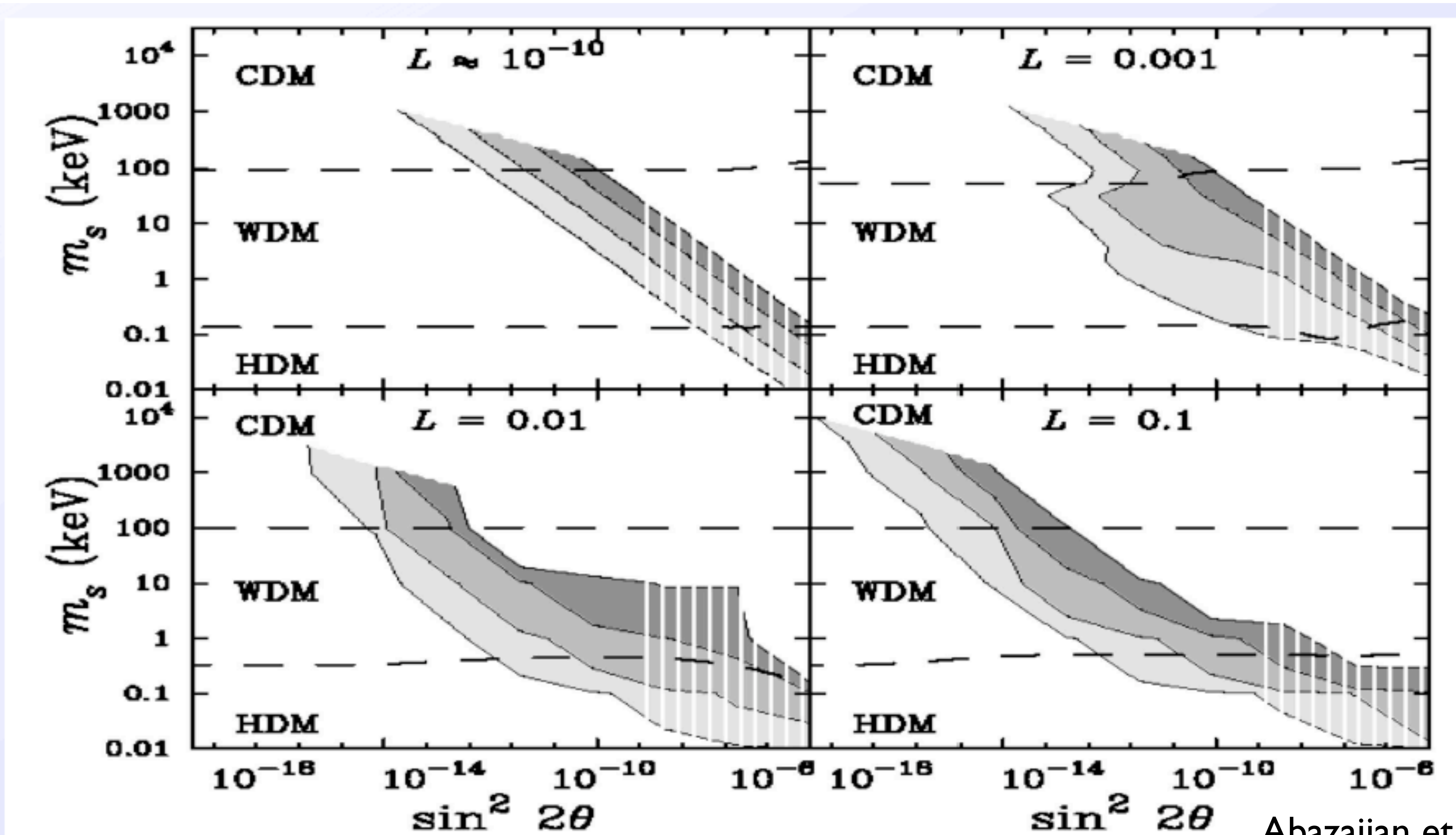
Fuller, Shi, 1998; Abazajian et al., 2001

The mixing angle is maximal, $\sin^2 2\theta_m = 1$, when the resonant condition is satisfied

$$\Delta(p)\cos 2\theta - \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 \mathcal{L} + |V_T| = 0$$

$$\left(\frac{m_4}{1\mathrm{keV}}\right)^2 \simeq 0.08 \frac{p}{T} \frac{\mathcal{L}}{10^{-4}} \left(\frac{T}{100\,\mathrm{MeV}}\right)^4 + 2\left(\frac{p}{T}\right)^2 \frac{B}{\mathrm{keV}} \left(\frac{T}{100\,\mathrm{MeV}}\right)^6$$

The production (both resonant and incoherent) is enhanced with respect to the case of negligible lepton asymmetry and much smaller values of the vacuum mixing angles are required.



Abazajian et al., 2001

Density matrix formalism

If more than one sterile neutrino is present, it is convenient to use the density matrix formalism: $\rho_{ij} = |\nu_i\rangle\langle\nu_j|$.

The Boltzmann equation can be rewritten as

$$\dot{\rho} = i[H_m + V_\alpha, \rho] - \{\Gamma, (\rho - \rho_{\mathrm{eq}})\}$$

- $H_m = U H_0 U^\dagger$
- $\{\Gamma, (\rho - \rho_{\mathrm{eq}})\}$ is responsible for the loss of coherence

$$\Omega_N h^2 = 7 \times 10^{-1} \frac{m_h^2}{10\ \mathrm{keV}^2} \sum_\alpha \frac{g_a}{\sqrt{C_a}} \frac{U_{\alpha j}^2}{10^{-8}}$$

The final abundance is the sum of the contribution of mixing with each active neutrino.

Melchiorri, et al., 2009

- Sterile neutrinos with small mixing angles and masses in the keV range are a viable dark matter candidate.

- They are “warm” as structures at very short distances do not develop.

- Different production mechanisms can be considered:
  - non-resonant neutrino oscillations
  - resonant neutrino oscillations
  - decays of heavier particles in extensions of the SM

- Searches are being carried out of their decays into photons.

Conclusions

- Neutrino oscillations have played a major role in the study of neutrino properties:
their discovery implies that neutrinos have mass and mix.

- They will continue to provide critical information as they are sensitive to the mixing angles, the mass hierarchy and CP-violation.

- A wide-experimental program is underway. Stay tuned!