Theory and Phenomenology of neutrino oscillations: Lecture I

GIF School
LES NEUTRINOS

APC Paris
12-16 September 2011

Silvia Pascoli
IPPP - Durham U.

What will you learn from these lectures?

- The basic picture of neutrino oscillations (mixing of states and coherence)

- The formal details: how to derive the probabilities

- Neutrino oscillations both in vacuum and in matter

- The implications of neutrino oscillations: neutrino masses and mixing

- Their relevance in present and future experiments

Useful references

- C. Giunti, C.W. Kim, Fundamentals of Neutrino Physics and Astrophysics, Oxford University Press, USA (May 17, 2007)
- C.W. Kim and A. Pevsner, Neutrinos in Physics and Atrophysics, Harwood academic publishers (1993)
- A. De Gouvea, TASI lectures, hep-ph/0411274
- S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987) 671
- S.M. Bilenky et al., Phys. Rept. 379 (2003) 69 [hep-ph/0211462]
- A. Strumia and F. Vissani, hep-ph/0606054.

Plan of lecture 1

- Brief history of neutrino oscillations
- Neutrino mixing
- Neutrino oscillations: the basic picture
- Neutrino oscillations: the details
- 2-neutrino oscillations
- 3-neutrino oscillations
- Subtle issues in neutrino oscillations

History of neutrino oscillations

- The first idea of neutrino oscillations was considered by B. Pontecorvo in 1957. [B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957)549. B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.]

- Mixing was introduced at the beginning of the '60 by Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28 (1962) 870, Y. Katayama, K. Matumoto, S. Tanaka, E. Yamada, Prog. Theor. Phys. 28 (1962) 675 and M. Nakagawa, et. al., Prog. Theor. Phys. 30 (1963)727.

- Few years later the first computation of the probability was performed [V. Gribov, B. Pontecorvo, Phys. Lett. B28 (1969) 493. See also B. Pontecorvo, Sov. Phys. JETP 26 (1967) 984].

First indications of ν oscillations came from solar ν.

- R. Davis built the Homestake experiment to detect solar ν, based on an experimental technique by Pontecorvo.

- Compared with the predicted solar neutrinos fluxes (J. Bahcall et al.), a significant deficit was found. First results were announced [R. Davis, Phys. Rev. Lett. 12 (1964)302 and R. Davis et al., Phys. Rev. Lett. 20 (1968) 1205].

- This anomaly received further confirmation by other experiments (SAGE, GALLEX, SuperKamiokande, SNO...) and was finally interpreted as neutrino oscillaions.

An anomaly was also found in atmospheric neutrinos.

- Atmospheric neutrinos had been observed by various experiments but the first relevant indication of an anomaly was presented in 1988 [Kamiokande Coll., Phys. Lett. B205 (1988) 416], subsequently confirmed by MACRO.

- The anomaly was interpreted as neutrino oscillations.

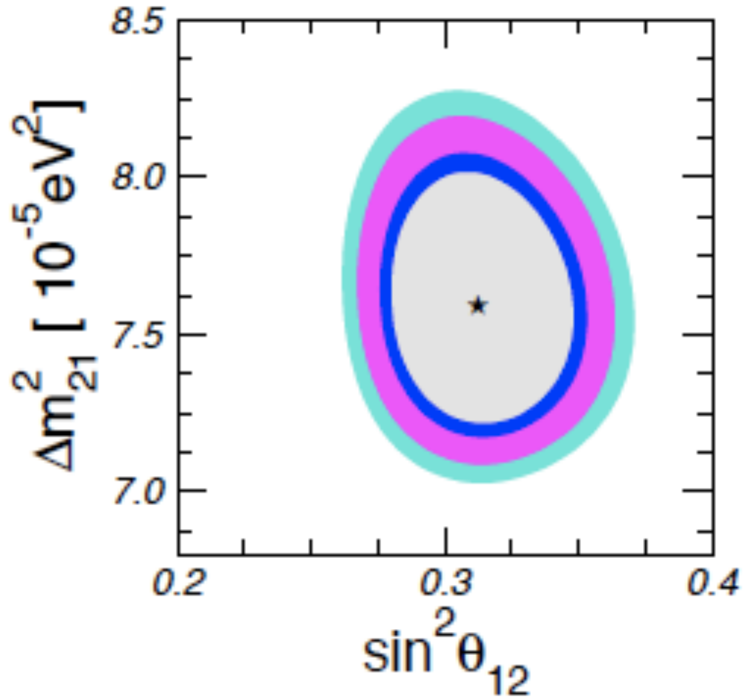
- Strong evidence was presented in 1998 by SuperKamiokande (corroborated by Soudan2 and MACRO) [SuperKamiokande Coll., Phys. Rev. Lett. 81 (1998) 1562]. This is considered the start of "modern neutrino physics"!

Since 1998, an impressive amount of data has been found of oscillations of solar, atmospheric, reactor and accelerator neutrinos.

SAGE, GALLEX

MACRO

KamLAND



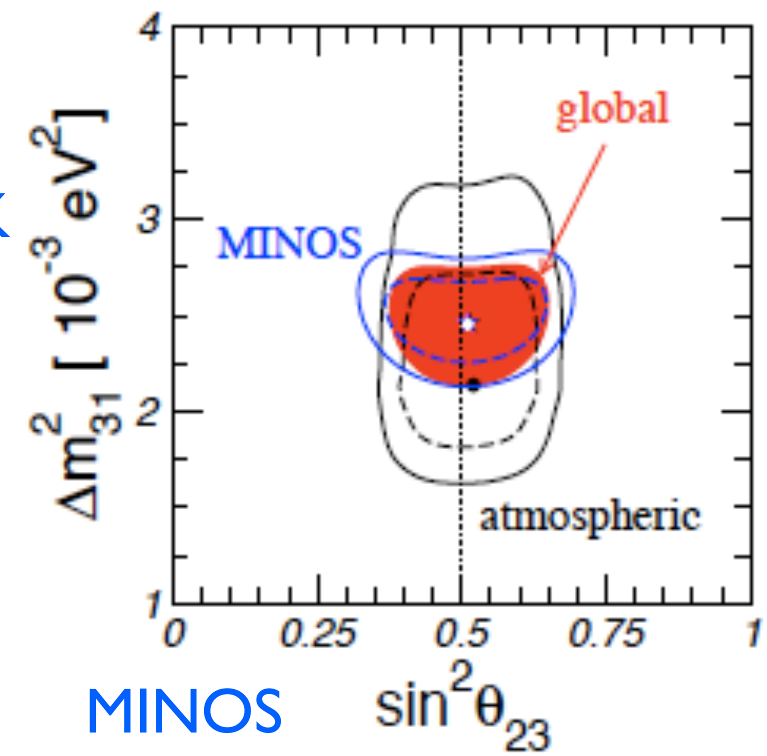
The solar sector

K2K

The atmospheric sector

Schwetz et al., 2011,
see also Gonzalez-Garcia and Maltoni

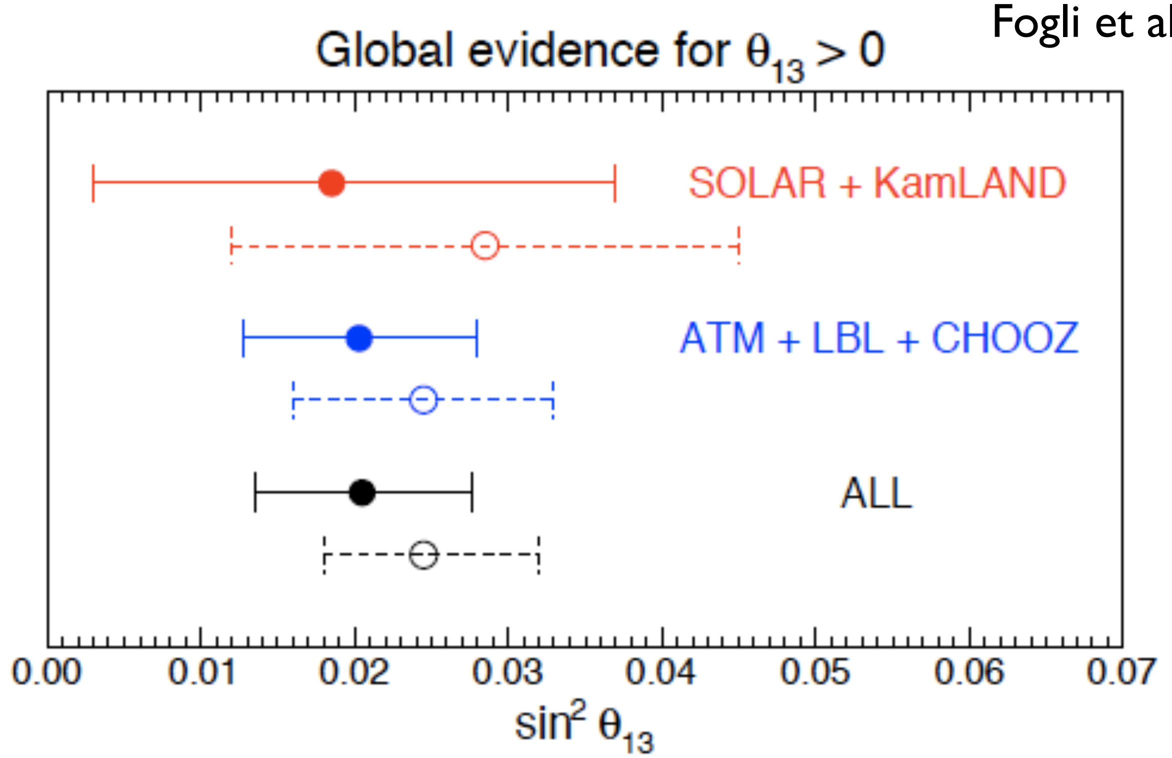
SNO



Super-Kamiokande

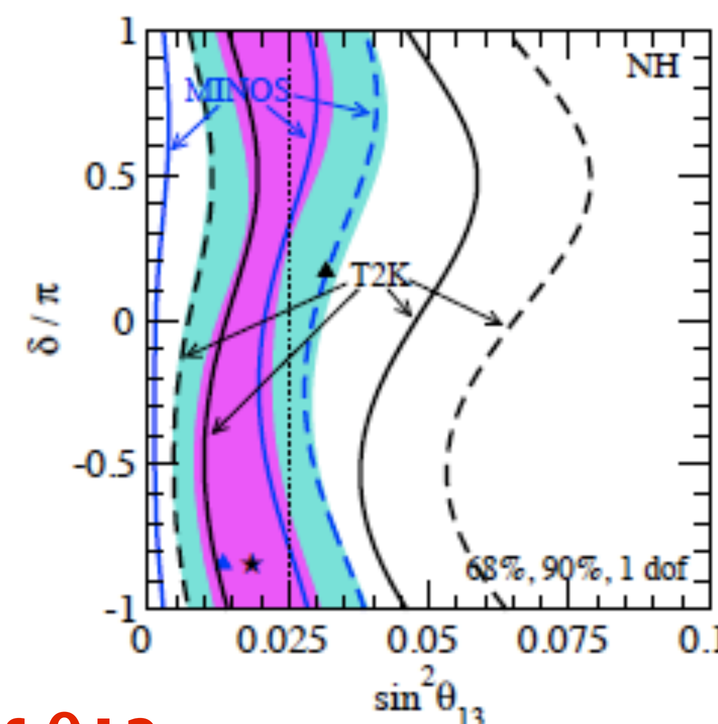
MINOS

Fogli et al., 2011



CHOOZ

T2K



Hints of θ13.

Schwetz et al., 2011,

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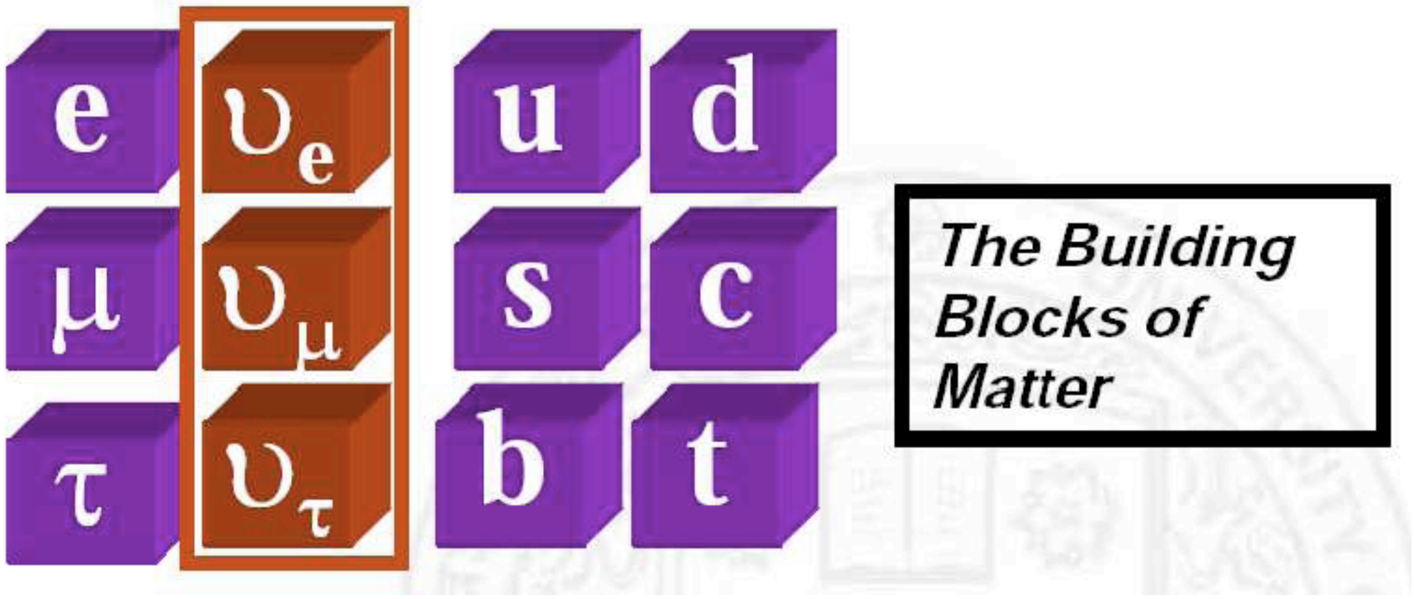
Neutrinos in the SM



- Neutrinos come in 3 flavours, corresponding to the charged lepton.

- They belong to SU(2) doublets:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- In the SM, neutrinos do not have mass. But we know that this is not the case.

Neutrino interactions

- They have charged current (CC) interactions

$$\mathcal{L}^{CC}_{\text{lept int}} = -\frac{g}{\sqrt{2}}\left(\sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L}\, \gamma_\rho\, l_{\alpha L}\, W^\rho + \text{h.c.}\right)$$

This implies that in an interaction in which a muon is produced, the corresponding (anti-)neutrino will be a muon (anti-)neutrino: $\pi^- \to \mu^- \bar{\nu}_\mu$.

- And neutral current ones.

$$\mathcal{L}^{NC}_{\text{lept int}} = -\frac{g}{4\cos\theta_W}\left(\sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha\, \gamma_\rho\, (1-\gamma_5)\, \nu_\alpha\, Z^\rho + \text{h.c.}\right)$$

Notice that this Lagrangian is invariant under a global U(1) transformation for each generation

$$l_\alpha \rightarrow e^{i\phi}\, l_\alpha \qquad \nu_\alpha \rightarrow e^{i\phi}\, \nu_\alpha$$

This is the so called family lepton number, L_i .

The global lepton number, L is $L = L_e + L_\mu + L_\tau$

$$L = \int d^3x \left[\sum_{k=1}^{3} \nu_k^\dagger(x)\nu_k(x) + \sum_{\alpha=e,\mu,\tau} l_\alpha^\dagger(x) l_\alpha(x) \right]$$

Lepton number is one of the important symmetries of particle interactions: it is related to the nature of neutrinos, the origin of neutrino masses and of the baryon asymmetry of the Universe.

Neutrino masses

Neutrinos have mass (see Smirnov's lectures):

$$\mathcal{L}_{\text{lepton}} = \mathcal{L}_{SM} + \text{neutrino masses}$$

For instance, a Majorana mass term

$$\mathcal{L}^{M}_{\text{mass}} = \frac{1}{2}\nu_L^T C^\dagger M_M \nu_L + \text{h.c.}$$

The matrix M typically will be non-diagonal in the flavour basis and can be diagonalised by a congruent transformation.

$$M_M = U_\nu^* \,\text{diag}(m_1, m_2, m_3)\, U_\nu^\dagger$$

Neutrino mixing

Mixing is described by the Pontecorvo-Maki-Nakagawa-Sakata matrix, which enters in the CC interactions

$$
\begin{aligned}
|\nu_\alpha\rangle &= \sum_i U_{\alpha i} |\nu_i\rangle \\
\mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}} \sum_{k\alpha} \left(U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.} \right)
\end{aligned}
$$

Note: the diagonalisation of the charged lepton Lagrangian can also contribute to U:

$$U = U_\nu \, U_\ell^\dagger$$

What we measure in neutrino oscillations is the combination U.

2-neutrino mixing matrix depends on 1 angle only. The phases get absorbed in a redefinition of the leptonic fields (a part from 1 Majorana phase).

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

3-neutrino mixing matrix has 3 angles and 1(+2) CPV phases.

$$\begin{pmatrix} \bar{\nu}_1 & \bar{\nu}_2 & \bar{\nu}_3 \end{pmatrix} e^{i\psi} \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{CKM-} \\ \text{type} \end{pmatrix} \begin{pmatrix} e^{i\rho_e} & 0 & 0 \\ 0 & e^{i\rho_\mu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

Rephasing

$$\begin{aligned} e &\to e^{-i(\rho_e+\psi)} e \\ \mu &\to e^{-i(\rho_\mu+\psi)} \mu \\ \tau &\to e^{-i\psi} \tau \end{aligned}$$

the kinetic, NC and mass terms are not modified: these phases are unphysical.

For Dirac neutrinos, the same rephasing can be done. For Majorana neutrinos, the Majorana condition forbids such rephasing: 2 physical CP-violating phases.

$$U = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar, reactor } \theta_{12} \sim 30^o} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atm, Acc } \theta_{23} \sim 45^o}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix}}_{\text{CPV phase}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}_{\text{React, Acc } \theta_{23} < 12^o} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_1} & 0 \\ 0 & 0 & e^{-i\phi_2} \end{pmatrix}}_{\text{CPV Majorana phases}}$$

For antineutrinos, $U \rightarrow U^*$

CP-violation

CP-symmetry is one of the important symmetries in particle physics. It is broken in the quark sector. Is there CP-violation also in the leptonic one?

Under a CP-transformation

$$U_{CP}\psi(x,t)U_{CP}^{-1} = \eta_k i\gamma^0 C\bar{\psi}^T(-x,t)$$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\sum_{k\alpha}\left(U^*_{\alpha k}\bar{\nu}_{kL}\gamma^\rho l_{\alpha L}W_\rho + U_{\alpha k}\bar{l}_{\alpha L}\gamma^\rho \nu_{kL}W^\dagger_\rho\right)$$

$$U_{CP}\mathcal{L}_{CC}U_{CP}^{-1} = -\frac{g}{\sqrt{2}}\sum_{k\alpha}\left(U_{\alpha k}\eta_k i\bar{\nu}_{kL}\gamma^\rho l_{\alpha L}W_\rho + U^*_{\alpha k}\eta^*_k i\bar{l}_{\alpha L}\gamma^\rho \nu_{kL}W^\dagger_\rho\right)$$

where η_k is the neutrino phase and we used

$$U_{CP}W^\rho U_{CP}^{-1} = -W^{\rho\dagger}$$

CP-conservation requires U is real $\Rightarrow \delta = 0, \pi$

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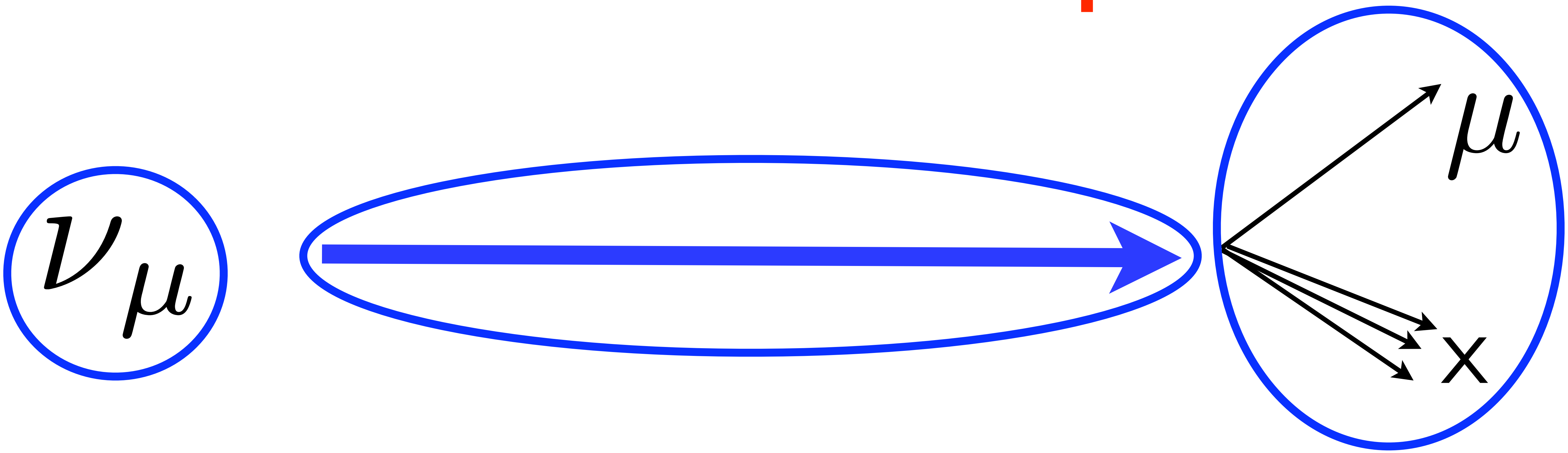
Neutrino oscillations: the picture

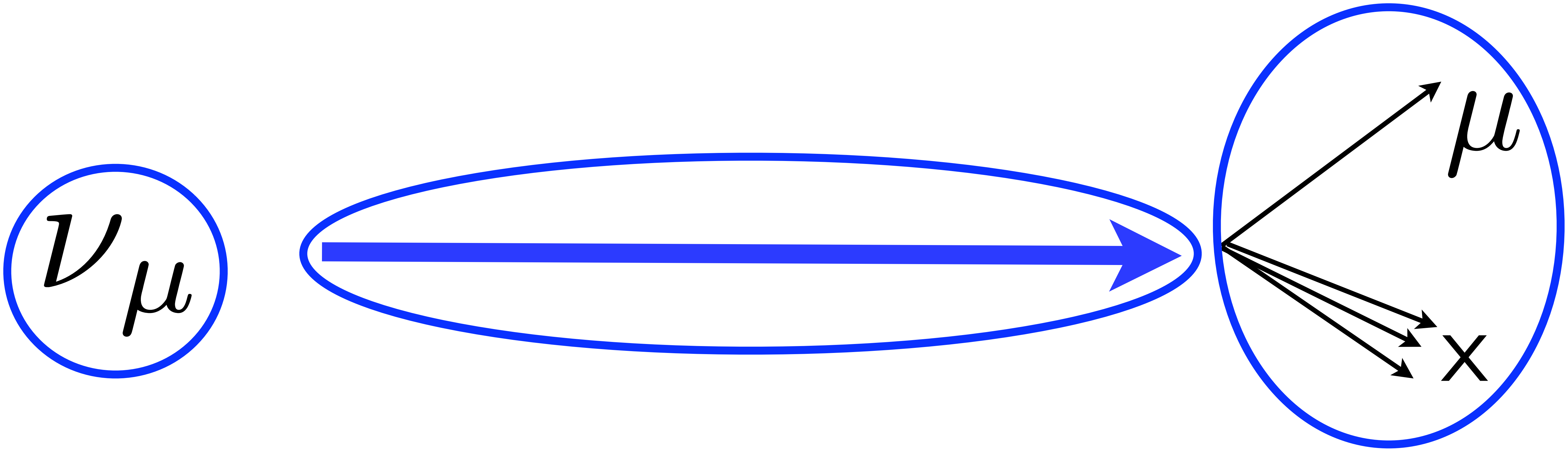


Production — Propagation — Detection

Flavour states — Massive states (eigenstates of the Hamiltonian) — Flavour states

At production, coherent superposition of massive states:

$$|\nu_\mu\rangle = U_{\mu 1}|\nu_1\rangle + U_{\mu 2}|\nu_2\rangle + U_{\mu 3}|\nu_3\rangle$$



Production

$$|\nu_\mu\rangle = \sum_i U_{\mu i} |\nu_i\rangle$$

Propagation

$$\nu_1 : \quad e^{-iE_1 t}$$

$$\nu_2 : \quad e^{-iE_2 t}$$

$$\nu_3 : \quad e^{-iE_3 t}$$

Detection: projection over

$$\langle \nu_\mu |$$

As the propagation phases are different, the state evolves with time and can change to other flavours.

Neutrino oscillations are analogous to many other systems in QM, in which the initial state is a coherent superposition of the eigenstates of the Hamiltonian:

- NH3 molecule: produced in a superposition of "up" and "down" states

- Spin states: for example a state with spin up in the z-direction in a magnetic field aligned in the x-direction B=(B,0,0). This gives raise to spin-precession, i.e. the state changes the spin orientation with a typical oscillatory behaviour.

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Neutrino oscillations: the theory
i) in vacuum

Let's assume that at t=0 a muon neutrino is produced

$$|\nu, t=0\rangle = |\nu_\mu\rangle = \sum_i U_{\mu i}|\nu_i\rangle$$

The time-evolution is given by the solution of the Schroedinger equation with free Hamiltonian:

$$|\nu, t\rangle = \sum_i U_{\mu i} e^{-iE_i t}|\nu_i\rangle$$

In the same-momentum approximation:

$$E_1 = \sqrt{p^2 + m_1^2} \quad E_2 = \sqrt{p^2 + m_2^2} \quad E_3 = \sqrt{p^2 + m_3^2}$$

Note: other derivations are also valid (same E formalism, etc).

At detection one projects over the flavour state as these are the states which are involved in the interactions. The probability of oscillation is

$$
\begin{aligned}
P(\nu_\mu \to \nu_\tau) &= |\langle \nu_\tau | \nu, t\rangle|^2 \\
&= \left| \sum_{ij} U_{\mu i} U^*_{\tau j} e^{-iE_i t} \langle \nu_j | \nu_i \rangle \right|^2 \\
&= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-iE_i t} \right|^2
\end{aligned}
$$

Typically neutrinos are very relativistic: $E_i \simeq p + \frac{m_i^2}{2p}$

$$
\begin{aligned}
&= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-i\frac{m_i^2}{2E}t} \right|^2 \\
&= \left| \sum_{i} U_{\mu i} U^*_{\tau i} e^{-i\frac{m_i^2 - m_1^2}{2E}t} \right|^2
\end{aligned}
$$

Δm^2_{i1}

Exercise

Implications of the existence of neutrino oscillations

Notation: α and β refer to flavour states; 1,2,3... to massive states.
Unless specified, $\alpha \neq \beta$.

The oscillation probability implies that

$$P(\nu_\alpha \to \nu_\beta) \quad = \quad \left| \sum_i U_{\alpha 1} U^*_{\beta 1} e^{-i \frac{\Delta m^2_{i1}}{2E} L} \right|^2$$

• **neutrinos have mass** (as the different components of the initial state need to propagate with different phases)

• **neutrinos mix** (as U needs not be the identity. If they do not mix the flavour eigenstates are also eigenstates of the Hamiltonian and they do not evolve)

General properties of neutrino oscillations

- Conservation of probability: $\sum_{\beta} P(\nu_\alpha \to \nu_\beta) = 1$

- Neutrino oscillations conserve the total lepton number:

a neutrino is produced and evolves with time

- They violate the flavour lepton number as expected due to mixing.

- Neutrino oscillations do not depend on the overall mass scale and on the Majorana phases.

Recall that for antineutrinos $U \rightarrow U^*$

- CPT invariance: $P(\nu_\alpha \rightarrow \nu_\beta) \;=\; P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$

$$\left| \sum_i U_{\alpha_i} U^*_{\beta i} e^{-iE_i t} \right|^2 = \left| \sum_i U_{\beta i} U^*_{\alpha i} e^{iE_i t} \right|^2$$

Exercise

- CP-violation

$P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ **requires** $U \neq U^*(\delta \neq 0, \pi)$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) \quad \text{as} \quad \left| \sum_i |U_{\alpha_i}|^2 e^{-iE_i t} \right|^2 = \left| \sum_i |U^*_{\alpha_i}|^2 e^{-iE_i t} \right|^2$$

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2-neutrino case

Let's recall that the mixing is

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

We compute the probability of oscillation

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} \right|^2 \\ &= \left| \cos\theta \sin\theta - \cos\theta \sin\theta e^{-i \frac{\Delta m^2_{21}}{2E} L} \right|^2 \\ &= \cos^2\theta \sin^2\theta \left| 1 - \cos\left(\frac{\Delta m^2_{21}}{2E} L\right) - i \sin\left(\frac{\Delta m^2_{21}}{2E} L\right) \right|^2 \\ &= \frac{1}{2} \sin^2(2\theta) \left(1 - \cos\left(\frac{\Delta m^2_{21}}{2E} L\right) \right) \\ &= \boxed{\sin^2(2\theta) \sin^2\left(\frac{\Delta m^2_{21}}{4E} L\right)} \end{aligned}$$

Exercise

The oscillation phase

$$\frac{\Delta m_{21}^2}{4E} L = 1.27 \; \frac{\Delta m_{21}^2[\mathrm{eV}^2]}{4 \; E[\mathrm{GeV}]} L[\mathrm{km}]$$

Exercise

Depending on E, L and Δm_{21}^2

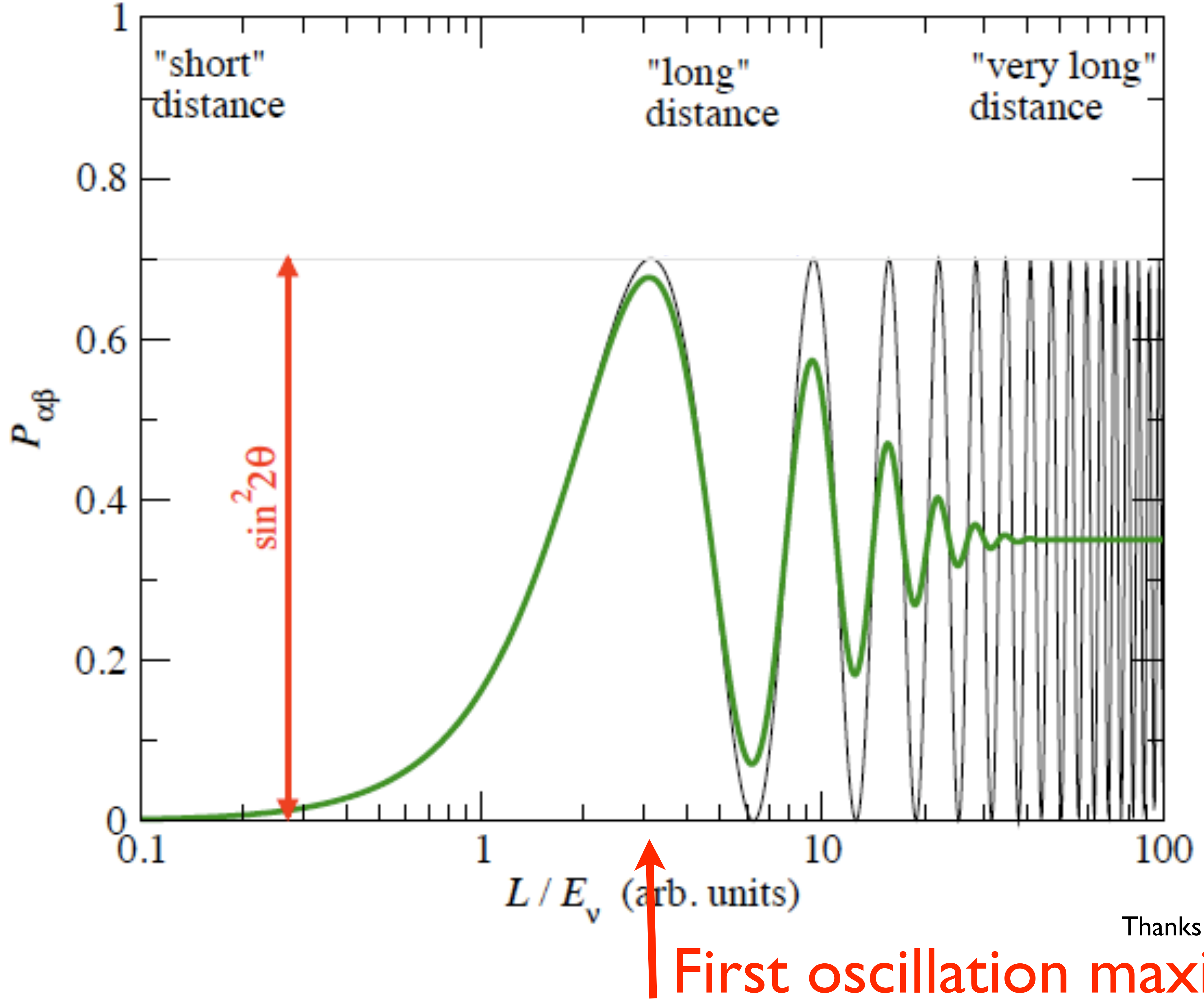
- $\frac{\Delta m_{21}^2}{4E} L \ll 1$: oscillations do not develop

$$P(\nu_\alpha \to \nu_\beta) \simeq 0$$

- $\frac{\Delta m_{21}^2}{4E} L \sim \mathcal{O}(1)$: oscillatory behaviour observed

- $\frac{\Delta m_{21}^2}{4E} L \gg 1$: oscillations are averaged out

$$P(\nu_\alpha \to \nu_\beta) \simeq \frac{1}{2} \sin^2(2\theta)$$



Thanks to T. Schwetz

Properties of 2-neutrino oscillations

- Appearance probability:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- Disappearance probability:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{21}}{4E}L)$$

- No CP-violation as there is no Dirac phase in the mixing matrix

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

- Consequently, no T-violation (using CPT):

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$$

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3-neutrino oscillations

They depend on two mass squared-differences

$$\Delta m^2_{21} \ll \Delta m^2_{31}$$

3 angles and one CPV phase

$$\theta_{12}, \theta_{23}, \theta_{13}, \quad \delta$$

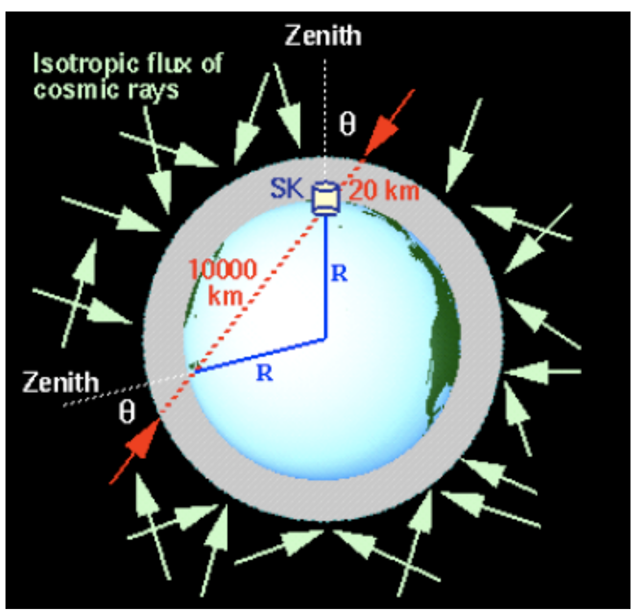
In general the formula is quite complex

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} e^{-i \frac{\Delta m^2_{21}}{2E} L} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

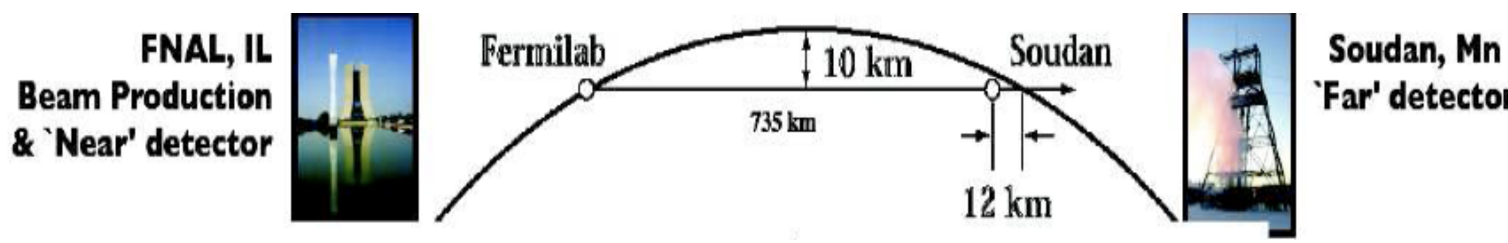
Interesting 2-neutrino limits

For a given L, the neutrino energy determines the impact of a mass squared difference. Various limits are of interest in concrete experimental situations.

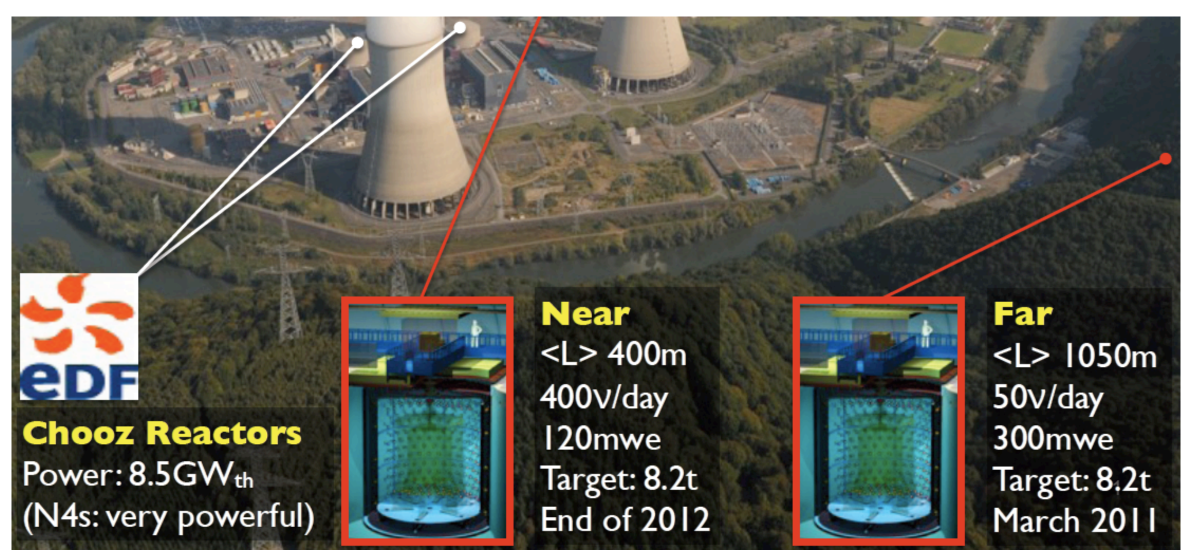
- $\frac{\Delta m^2_{21}}{4E} L \ll 1$, applies to atmospheric, reactor (CHOOZ...), current accelerator neutrino experiments



L~10000 km, E~1-10 GeV



L= 735 km, E~4 GeV



L~ 1 km, E~3 MeV

Thanks to DC collaboration (Cabrera)

The oscillation probability reduces to a 2-neutrino limit:

$$P(\nu_\alpha \to \nu_\beta) = \left| U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

We use the fact that $U_{\alpha 1} U^*_{\beta 1} + U_{\alpha 2} U^*_{\beta 2} + U_{\alpha 3} U^*_{\beta 3} = \delta_{\alpha\beta}$

$$= \left| -U_{\alpha 3} U^*_{\beta 3} + U_{\alpha 3} U^*_{\beta 3} e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

$$= \left| U_{\alpha 3} U^*_{\beta 3} \right|^2 \left| -1 + e^{-i \frac{\Delta m^2_{31}}{2E} L} \right|^2$$

The same we have encountered in the 2-neutrino case

$$= 4 \left| U_{\alpha 3} U_{\beta 3} \right|^2 \sin^2 \left(\frac{\Delta m^2_{31}}{4E} L \right)$$

Exercise

In terms of mixing angles we have

$$P(\nu_\mu \to \nu_e; t) = s_{23}^2 \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\tau; t) = c_{13}^4 \sin^2(2\theta_{23}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

$$P(\nu_\mu \to \nu_\mu; t) = 1 - 4 s_{23}^2 c_{13}^2 (1 - s_{23}^2 c_{13}^2) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

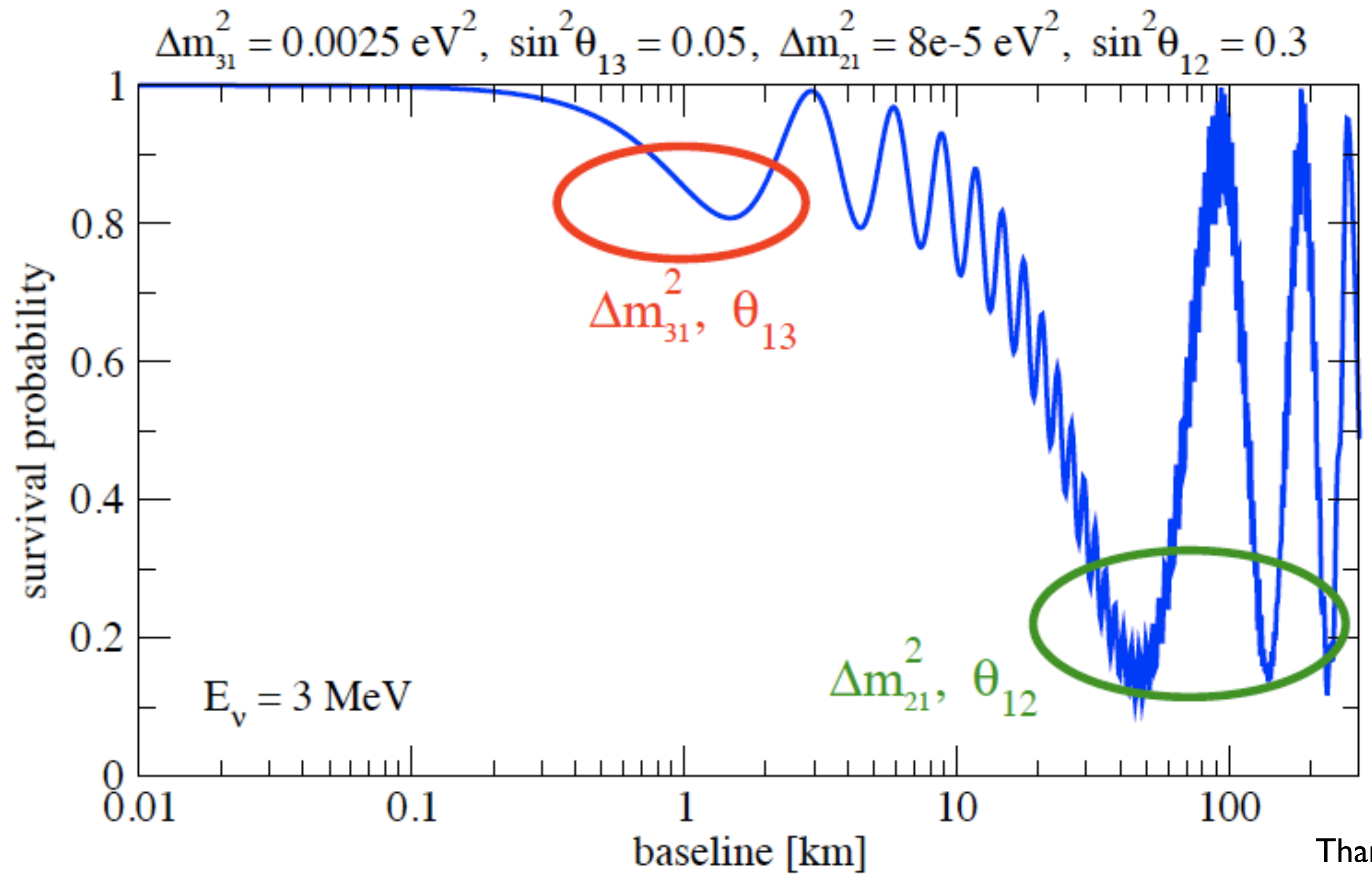
$$P(\nu_e \to \nu_e; t) = 1 - \sin^2(2\theta_{13}) \sin^2 \frac{\Delta m_{31}^2 L}{4E}$$

Exercise

- $\frac{\Delta m^2_{31}}{4E} L \gg 1$: for reactor neutrinos (KamLAND).

The oscillations due to the atmospheric mass squared differences get averaged out.

$$P(\bar{\nu}_e \to \bar{\nu}_e; t) \simeq c_{13}^4 \left(1 - \sin^2(2\theta_{12}) \sin^2 \frac{\Delta m^2_{21} L}{4E}\right) + s_{13}^4$$



Thanks to T. Schwetz

CP-violation will manifest itself in neutrino oscillations, due to the delta phase. Let's consider the CP-asymmetry:

$$P(\nu_\alpha \to \nu_\beta; t) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta; t) =$$

$$= \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} + U_{\alpha 3} U_{\beta 3}^* e^{-i \frac{\Delta m_{31}^2 L}{2E}} \right|^2 - (U \to U^*)$$

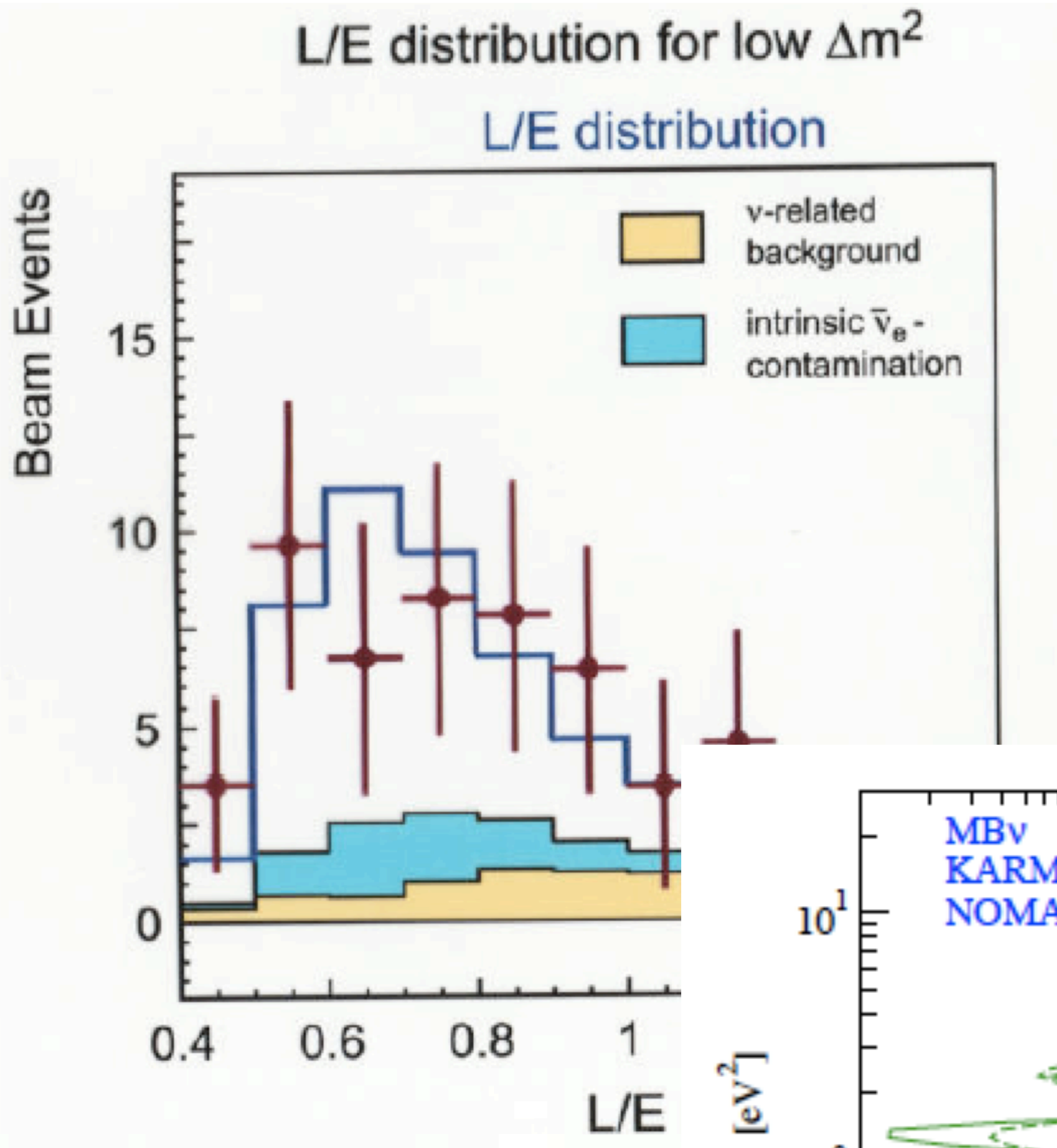
$$= U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2} e^{i \frac{\Delta m_{21}^2 L}{2E}} + U_{\alpha 1}^* U_{\beta 1} U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} - (U \to U^*) + \cdots$$

$$= 4 s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin\delta \left[\sin\left(\frac{\Delta m_{21}^2 L}{2E} \right) + \sin\left(\frac{\Delta m_{23}^2 L}{2E} \right) + \sin\left(\frac{\Delta m_{31}^2 L}{2E} \right) \right]$$

Exercise

- CP-violation requires all angles to be nonzero.
- It is proportional to the sine of the delta phase.
- If one can neglect Δm_{21}^2, the asymmetry goes to zero as we have seen that effective 2-neutrino probabilities are CP-symmetric.

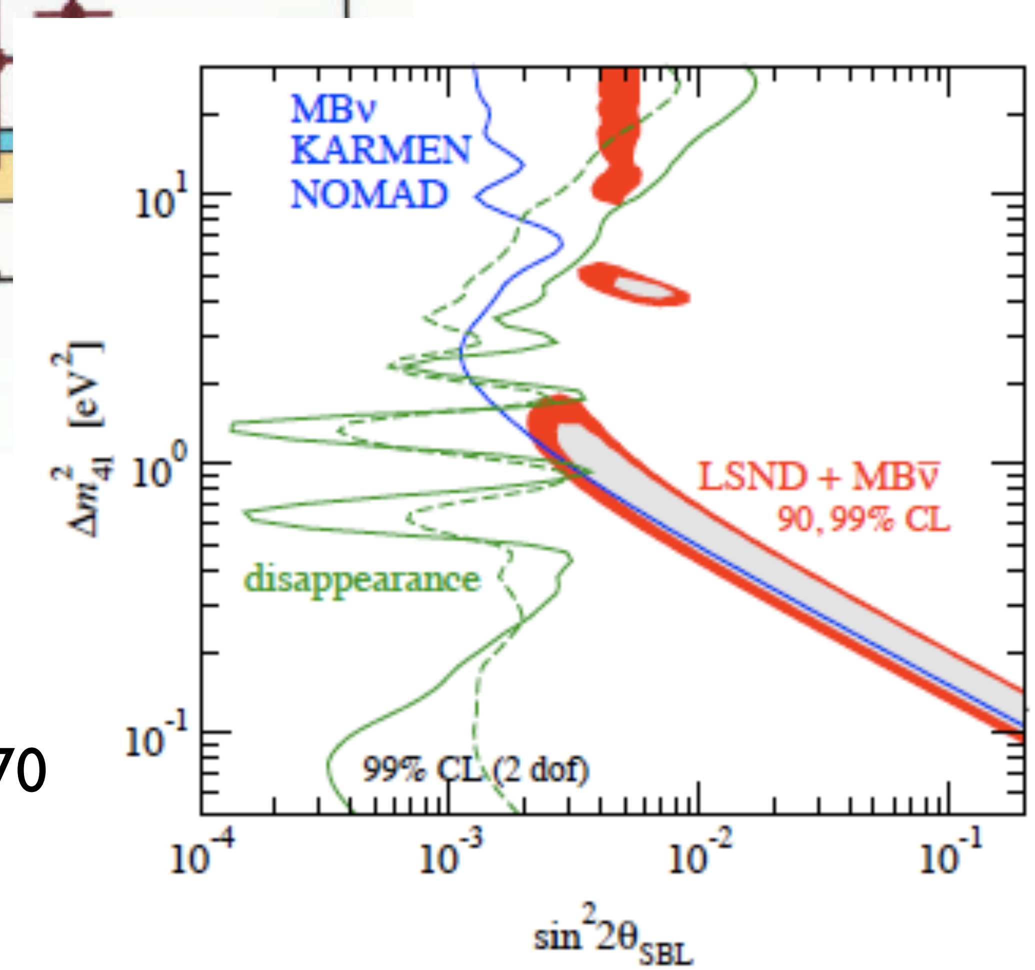
4- or 5- neutrino oscillations: sterile neutrinos



LSND

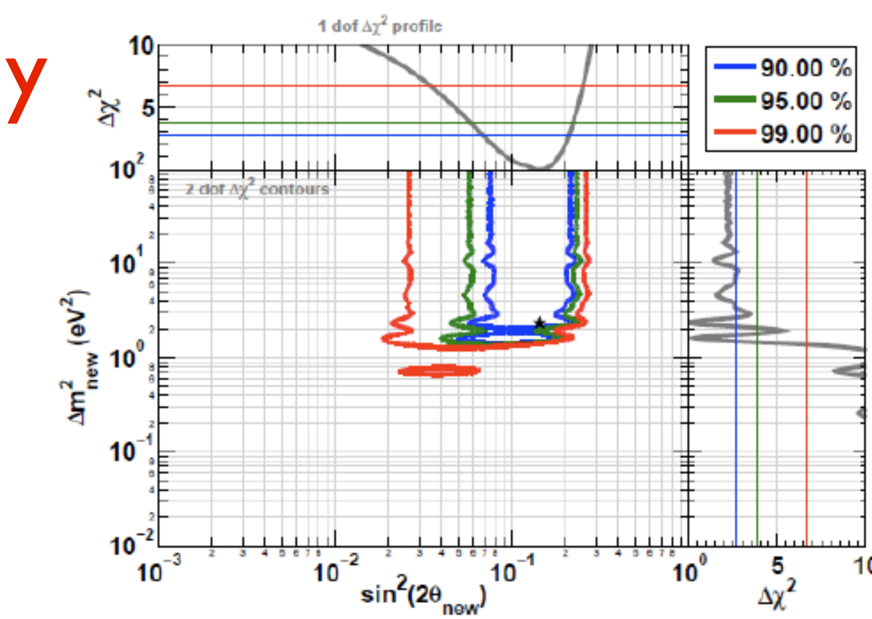
Kopp et al., 1103.4570

MiniBooNE



Reactor anomaly

Combining reactor rates + shape + Gallium Anomaly



no-oscillation disfavored at 99.8% CL

G. Mention et al., 1101.2755

Various hints of oscillations with

$$\Delta m^2 \sim 1 \text{ eV}^2$$

As the Δm^2 required to explain these experiments is different from $\Delta m^2_{\rm sol}$ and $\Delta m^2_{\rm A}$, this means that there are at least 4 neutrinos. The fourth one needs to be sterile. The 2-neutrino limit applies.

Disappearance experiments

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e4}|^2(1 - |U_{e4}|^2)\sin^2(\Delta m^2 L/4E)$$
$$P(\nu_\mu \to \nu_\mu) = 1 - 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2)\sin^2(\Delta m^2 L/4E)$$

Appearance experiments

$$P(\nu_\mu \to \nu_e) = 4|U_{e4}|^2|U_{\mu 4}|^2\sin^2(\Delta m^2 L/4E)$$

There is a tension between evidence in appearance and constraints from disappearance. (see Lasserre's and Rubbia's talks)

Plan of lecture 1

- Brief history of neutrino oscillations
- Neutrino mixing
- Neutrino oscillations: the basic picture
- Neutrino oscillations: the details
- 2-neutrino oscillations
- 3-neutrino oscillations
- **Subtle issues in neutrino oscillations**

Further theoretical issues on neutrino oscillations

Energy-momentum conservation

Let’s consider for simplicity a 2-body decay: $\pi \to \mu \, \bar{\nu}_\mu$.

Energy-momentum conservation seems to require:

$$E_\pi = E_\mu + E_1 \quad \text{with } E_1 = \sqrt{p^2 + m_1^2}$$
$$E_\pi = E_\mu + E_2 \quad \text{with } E_2 = \sqrt{p^2 + m_2^2}$$

These two requirements seem to be incompatible. Intrinsic quantum uncertainty and localisation of the initial pion lead to an uncertainty in the energy-momentum and allow coherence of the initial neutrino state.

- If the energy and/or momentum of the muon is measured with great precision, then coherence is lost and only neutrino $\nu 1$ (or $\nu 2$) is produced.

- In any typical experimental situation, this is not the case and neutrino oscillations take place.

- However for large mass differences, e.g. in presence of heavy sterile neutrinos, this situation could arise.

For a detailed discussion see, Akhmedov, Smirnov, 1008.2077.

The need for wavepackets

- In deriving the oscillation formulas we have implicitly assumed that neutrinos can be described by plane-waves, with definite momentum.

- However, production and detection are well localised and very distant from each other. This leads to a momentum spread which can be described by a wave-packet formalism.

Typical sizes:

- e.g. production in decay: the relevant timescale is the pion lifetime (or the time travelled in the decay pipe),

$$\Delta t \sim \tau_\pi \Rightarrow \Delta E \Rightarrow \Delta p \quad \Delta x$$

For details see, Akhmedov, Smirnov, 1008.2077; Giunti and Kim.

Decoherence and the size of a wave-packet

- The different components of the wavepacket, $\nu 1$, $\nu 2$ and $\nu 3$, travel with slightly different velocities (as their mass is different).

- If the neutrinos travel extremely long distances, these components stop to overlap, destroying coherence and oscillations.

- In terrestrial experimental situations this is not relevant. But this can happen for example for supernovae neutrinos.

What have we learnt today?

- We have looked at the basic picture of neutrino oscillations and how to compute neutrino oscillation probabilities in vacuum for 2- and 3- neutrino mixing.

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta) \sin^2 \frac{\Delta m^2 L}{4E}$$

- We have discussed the properties (including CP-violation) and implications of neutrino oscillations.

- We have mentioned some more subtle issues in neutrino oscillations (mass, mixing and coherence).

Plan for tomorrow

- Neutrino oscillations in matter

- Implications of neutrino oscillation information on neutrino properties

- Neutrino oscillations in experiments: a phenomenological perspective

- Neutrino oscillations in cosmology: the example of sterile neutrinos